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# Tests for Structural Breaks in Time Series Analysis: A Review of Recent Development

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## Abstract

*The issue related to a structural break or change point in the econometric and statistics literature is relatively vast. In recent decades it was increasing, and it got recognized by various researchers. The debates are about a structural break or parameter instability in the econometric models. Over some time, there has been a different mechanism, and theoretical development stretching the fundamental change and strengthen the econometric literature. Estimation of structural break has undergone significant changes. Instead of exploring the presence of a known structural break, now the emphasis is on tracing multiple unknown cracks using dynamic programming. The paper an attempt has been made to review the different forms of the presence of structural break(s) over the past.*

**Keywords:** Structural Breaks, Chow, Quandt-Andrews and Bai-Perron Tests.

## Introduction

There is a lot of development in time series econometrics in recent times. The following studies have chosen an extensive period series data and covered by various economic factors. Predominantly it was brought to a significant amount of attention in terms of theoretical and empirical verification in the field of econometrics (see, for example, Chow, 1960; Quandt, 1960; Andrews, 1993; Andrews and Ploberger, 1994; Hansen, 1997; Bai, 1997; Bai and Perron, 1994, 1997, 1998, 2003, 2003a and 2006 and Perron, 2005 among others). In general, the structural breaks can be recognized in the structure of the economy and ongoing policies; specifically in its timing, trend, change point and date shifts (probably one break and the recent debates on multiple breaks). The present paper dived into four sections. The first section outlines the overview of a structural break, the second section deals with the different approaches followed over the year; test for known breakpoints, test for unknown breakpoints and test for unknown multiple breakpoints. Finally, issues and discussion follow. The studies on structural break began with the work of Gregory Chow in 1960. Since 1960, the initiation of a fundamental break mechanism started. For the first time, a known structural break has done predicted by Gregory Chow (1960). It is a test of equality in the coefficients of the parameters of regression, and there is a breakpoint mechanism. Simultaneously, an analysis of unknown structural change has carried by Quandt (1960). He discussed the constant-coefficient against alternative with changes in the error variance.

However, during the second half of the 1970s – Brown, Durbin and Evans proposed the techniques to analyze recursive residuals using CUSUM test.

Chow test with multiple regimes and it covered less than  $k$  subsamples, which were done by Dufour (1982). Simultaneous equations of chow test were conducted by Lo and Newey (1985). Andrews and Fair (1988) studied general nonlinear models with analysis of variance test. Quandt test with simple linear regression models (intercept changes with alternative and intercept and slope changes) Kim and Siegmund (1989). Quandt test has been extended to linear regression with lagged dependent variables by Ploeger, Karmer, and Alt (1988, 1989). Linear regression with serially correlated errors was reviewed by Kao and Ross (1992). Canterll et al. (1991); Dufour, Glyssels, and Hall (1994) applied the general nonlinear models with the predictive test. Hansen (1995) provided detail p-values. Overall, it has been allocated for the following formation:

- Check for known breakpoints
- Test for unknown breakpoints
- Test for unknown multiple breakpoints

The present paper is performed in the following manner. The second section describes the traditional (Chow test) experiment with known breakpoints. The third section elaborates the analysis of unknown structural breaks, and the fourth section deals with multiple strange structural breaks. Finally, the discussion follows.

### Test with Known Breakpoints

In general, there is a sense to identify the breakpoints which are based on exogenous occurrence/outcomes (for instance, the Great Recession, Oil Shock, Liberalisation, Global Financial Crisis, and Eurozone Crisis) or arbitrary dates. Chow (1960) test is a test for a known break. It was thought about; tests of agreement between sets of coefficients in two long regressions. In that, he focused economic relationship in the linear regression model. For empirical verification, there were subjects that arise often. Such as: is the consumption pattern of the American people today the same as it was before World War II? Whether the relationship of two periods is the same or whether the same relationship holds for two different groups of units. He employed a formal model is that

$$\begin{aligned} y_1 &= X_1\beta_1 + \varepsilon_1 \dots\dots\dots 1 \\ y_2 &= X_2\beta_2 + \varepsilon_2 \dots\dots\dots 2 \end{aligned}$$

Where  $Y_1$  and  $Y_2$  and  $\varepsilon_1$  and  $\varepsilon_2$  are column vectors with  $n$  elements,  $X$  is a non-singular matrix, and  $\beta$  is the column vector of the  $\rho$  regression coefficients. The Chow (1960) test for a structural break and the further procedure is Fisher statistics (1970). However, it is considered being stationary (parameters are constant over time) with a single break and a statement about parameters. Subsequently, this was followed in linear regression with  $k$  observations and vector of  $n_1$  and  $n_2$  by using ordinary  $F$  test respectively. The Chow test has a long history, this could be obtained in different ways but widely used approaches stated by Rao (1952), Kullback and Rosenblatt (1957), Rao (1965) and Dufour (1982). First, the Chow test was associated with the analysis of variance test that is  $n_1 > k$ ,  $n_2 > k$ . Subsequently, it is used to find parameter constancy. Second, it is the predictive test that is  $n_1 > k$ ,  $n_2 < k$ , basically predictive test does not find the stability of the coefficient, but it does find the unbiasedness of  $n_1$  and  $n_2$ . Third, is the fundamental theorem of least squares test and it has extended multiple regimes with  $k$  samples. Subsequently, some subsample is covered less than knot all. However, Maddala (1998) stated that the motivation behind the chow test was to extend the analysis of variance test and the predictive test.

$$\left[ \frac{(RSS_s - RSS_T)/(n - k - n_1 + rk)}{RSS_T/(n_1 - rk)} \right] \dots\dots\dots (3)$$

RSS denotes subsamples Residual Sum of Squares, RSST denotes the total residual sum of squares from the regression, which is  $n_j > k$  and  $r \leq J$  then  $RSS/\sigma^2$  follows  $\chi^2$  with a degree of freedom  $(n - k)$  and  $RSST/\sigma^2$  follows  $\chi^2$  with a degree of freedom  $(n_1 - RK)$ . Let  $n = \sum n_j$  subsamples ( $j=1, 2, \dots, J$ ), 'r' consider the breaks, let  $n_1 = \sum n_j$  and it has  $F$ -degrees of freedom  $(n - k - n_1 + rk)$ ,  $(n_1 - rk)$ .

Numerous studies have brought the predominant assumption that the variance of two regressions or between two regimes could differ in the Chow test<sup>1</sup>. Many studies are being considered the testing

<sup>1</sup> MacKinnon (1989) identified that following authors discussed the issue of variance differs in the two sample periods; Toyoda (1974), Jayatissa (1977), Schmidt and Sickles (1977), Watt (1979), Hoda (1982), Phillips and McCabe (1983), Ohtani and Toyoda (1985), Toyoda and Ohtani (1986) and Weerahandi (1987). However, none of these studies have not proposed new approach to treat structural changes.

of fundamental breakthrough linear regression, including chow test. Toyoda (1974) has empirically verified the accuracy of the chow test under the conditions of heteroskedasticity in which any one of the sample sizes is very large. The level of significance test could affect as two sample sizes remain smaller. Under heteroscedasticity, the level of significance always becomes larger. Subsequently, Schmidt and Sickles (1977) investigated the accuracy and evidence of a chow test with Toyoda mechanism. They concluded that Toyoda finding somehow was inaccurate, and two sample sizes and the variances are different in each. Lo and Newey (1985) and Park (1991) implied an analysis of variance test to simultaneous equations. Andrews and Fair (1988) have adopted the analysis of variance test to general nonlinear econometric models. They have introduced a Wald test, Lagrange Multiplier-like test, and Likelihood Ratio test. Their results have shown a weak regulatory condition of heteroskedasticity. Dufour, Glyssels, and Hall (1994) had approached predictive analysis for structural stability and extended general nonlinear dynamic simultaneous equation models. This study also accounted for large subsample before the structural break; subsequently, structural changes in the second part are unknown. Recently Hansen (2001) empirical exploration stated that what would happen if parameters change in the fundamental change: dating breaks in the United States labor productivity. He used the data from February 1947 to April 2001, and he employed a simple first-order autoregressive dynamic model and focused on three aspects. Firstly, he focused on the unknown timing of the structural break. Secondly, an estimation of the timing of structural breaks and finally distinguishes between random walk and broken time trend. In this investigation, he found that there was substantial evidence of structural break between 1992 and 1996 and weaker evidence of the structural break in the 1960s and early 1980s. Hansen confined that the Chow test has two choices; first was the arbitrary break date and second was endogenous break date. In that there are limitations: first, the Chow test could be uninformative and exact break date might not be identified. The second statement of endogenous break date may have a chance of correlated with data, albeit Chow test could be misleading and break date

falsely exists before unknown features. However, there was a different answer which was observed in similar break dates, when he chose 1973 as a break date which had no structural break and then there was a structural break in 1975. Because of different answers, he treated structural breaks as unknown, for that he suggested Quandt test. However, in recent years, various authors have solved the problem and given a practical solution to the Quandt test.

### Test of Unknown Structural Breaks

The late 1970s, the literature on structural break were directed towards the detection of parameter instability, or parameter changes occurred at an unknown time. It emphasis particularly on parameters instability in dynamic models with trending regressors, co-integrated variables, heteroskedasticity disturbances, and perhaps Unit root (Bai 1993). There are various studies which focused on theoretical and empirical specification, those started with Quandt (1960), Farely and Hinich (1970), Brown *et al.* (1975), Ploberger (1983), Ploberger and Kramer (1990;1992), Perron (1989; 1991), Andrews (1990), Zivot and Andrews (1992), Hansen (1992), Chu and White (1992), Bai, Lumsdaine and Stock (1991), Banerjee, Lumsdaine and Stock (1992) and Christiano (1992). Testing of unknown structural break or change points can be identified in single and multiple structural breaks. When the structural break is known, then the Chow test is more powerful. Subsequently, when the structural breaks are unknown, it does not require any prior knowledge about the structural change which appeared in its timing, type, and shift. The major contribution of the unknown structural break is discussed below in details. The synoptic views of the single unknown structural break were compiled in the chapter, which was contributed by Vilares J (1986); Stock J.H and Watson (2010) and Maddala (1998). Vilares J (1986) classified that there are three important tests on a single unknown structural break. The number of studies which employed the general structure that is

$$= \left\{ \begin{array}{ll} X_{1t}\beta_1 + \varepsilon_{1t} & \text{if } t \leq 0 \\ X_{2t}\beta_2 + \varepsilon_{2t} & \text{if } t > 0 \end{array} \right\} \dots\dots\dots(4)$$

Where time  $t = 1, 2, \dots, T$  ( $t=0=Z_0=zt$ ) and the

explanatory variable of both the subset are same that is  $x_{1t} = x_{2t} = x_t$  therefore  $k_1 = k_2 = k$ .

The first and foremost modified version of the Chow test is called Quandt Likelihood Ratio Statistics. Quandt (1960) employed the tests of the hypothesis that a linear regression system obeys two separate regimes. In that, the entire observation was split into two subsets. Over the entire samples, the unknown time  $m$  observation predicted from one regime and forwarding observations come from the other subset. However, Quandt took the initiative of change point analysis in a time-varying regression model. This mechanism was applied based on Switching Regression Model (SRM) and Quantity Rationing Models (QRM). This test procedure was considered to predict the significance of the maximum value of the likelihood ratio statistic while employing recursive switching models. It can be considered basing on Max Chow test. General formulation of Switching Regression Model is one of the deterministic assignments which is as follows

$$Y_t = \begin{cases} x_{1t}\beta_1 + \varepsilon_{1t} & \text{if } z_t \pi < 0 \\ x_{2t}\beta_2 + \varepsilon_{2t} & \text{if } z_t \pi \geq 0 \end{cases} \dots \dots \dots (5)$$

Where it is a row vector of  $p$  variables of known constant (which may belong to  $X_{1t}$  and  $X_{2t}$ )  $\pi$  is a column vector of  $p$  unknown constant. The explanatory variable of both the subset is the same that is  $x_{1t} = x_{2t} = x_t$ . Quandt model has two sets of functions that are

$$\ln L(t) = -\frac{T}{2} \ln 2\pi - \frac{1}{2} (Y - X\beta)' (X'X)^{-1} (Y - X\beta) = -\frac{1}{2} (Y - X\beta)' (X'X)^{-1} (Y - X\beta) \dots \dots \dots (6)$$

Where  $\alpha$  is a vector of an estimated parameter ( $\beta_1', \beta_2', \sigma_1, \sigma_2$ ) in  $2k+2$ , the above function has allowed calculating the estimated  $\hat{\alpha}$  for  $\alpha$  that is a function of  $t_0$ :  $\hat{\alpha} = \hat{\alpha}(t_0)$ . The other is above function  $\alpha$  has replaced by  $\hat{\alpha}$  the result function has  $t_0$  to replaced by  $t$ .

$$\text{The likelihood ratio function is } R = \frac{\hat{\sigma}_1^2 \hat{\sigma}_2^2 (T - t_0)}{\hat{\sigma}^2} \dots \dots \dots (7)$$

$T = 1, 2, \dots, T$  total sample of the regression model and  $\sigma^2$  has estimated variance. However,  $t_0$  is not a continuous variable than the calculation of the likelihood ratio become problematic.

More interestingly the critical value of the QLR statistics provided more extensive distribution than

F-statistics, and it would be applicable for a larger sample. It indicates that the QLR statistics has an immense majority of rejecting the null hypothesis rather F-statistics, while there are multiple discrete breaks which were encountered. Subsequently, the first trimming mechanism was offered by Quandt 1960; the critical value of QLR statistics trimming a standard range of 15 percent. Kim and Siegmund (1989) proposed likelihood ratio tests to detect a change point (broken line) in a simple regression model. They had started with the question of when the alternative specifies that only the intercept changes or option permits both the intercept and slope changes. Approximation for the significance level in the model allowed intercept change. With the help of the inversion of the likelihood ratio tests and model intercepts and slopes can be obtained through the confidence region and joint confidence region for the change point. Given  $y_i = \alpha_0 + \beta_0 x_i$  ( $i = 1, \dots, m$ ), if the change point  $j$  ( $i \leq j$ ) and an approximate joint confidence region for  $j$  ( $i > j$ ), the difference  $\alpha_0 - \alpha_1$ , and  $\beta$ ; to select model between change point without covariates and then model without change point. Kim and Siegmund stated that their procedure probably is satisfactory if the  $x$ 's are random and there is a loss of accuracy in estimating  $\alpha_0 - \alpha_1$ , empirically caused by  $\beta$  and  $x_i$  for  $1 \leq j$  and  $m > j$  which has disjointed support. Subsequently, Maddala (1998) pointed out the above procedure, which was hindered by the lack of distribution theory. Porrier (1976) stated that the appropriate likelihood ratio function does not hold their standard regularity condition and Quandt distribution of  $2 \ln R$  observed a sparse approximation.

During 1970 FH test, which was proposed by Farley and Hinich (1970), can also be applied to the general formation model (2). Their primary assumption was considered as ( $t = 1, 2, \dots, T$ ) which had an equal chance of occurrence switch point  $t_0$  to each  $t$  observation.

When time  $t_0$  was known than the model (2) had framed follows

$$y_t = x_t \beta_1 + v_t \delta + \varepsilon_t \dots \dots \dots (8)$$

Where time  $t = 1, 2, \dots, T$ ,  $v_t$  is a vector of  $(1 \times k)$  and  $\delta = \beta_2 - \beta_1$ . However,  $t_0$  was not known then FH test can be written (instead of  $v_t$  by  $x_t$ )

$$y_t = x_t (\beta_1 + \delta t) + \varepsilon_t \dots \dots \dots (9)$$

The above procedure applied in the model (2) to test the null hypothesis, then  $\delta$  is equal to 0. In general, the validity of the FH test probably identifies the general information than the exact specification of  $t_0$  information by Vilares J (1986). Farely *et al.* (1975) suggested pseudo chow test is also useful. In that, they assumed the breakpoint  $t_0$  could have occurred midpoint of  $t$ , and the above procedure was applied. Poirier (1976) used the Monte-Carlo technique to begin Investigation and also tested the above three procedures. Basing on which he concluded that the likelihood ratio does not determine its result. However, when the breakpoint is wider, or the sample is larger than the above three tests were not applicable, which were stated by Maddala (1998).

CUSUM and CUSUM of Squared test were one of the essential classical tests which were suggested by Brown *et al.* (1975). They applied recursive residuals to test single structural break over time while parameters changed.

$$CUSUM_t = \sum_{j=k+1}^t \frac{\hat{w}_j}{\hat{\sigma}_w} \quad \dots \dots \dots (10)$$

$$\hat{\sigma}_w^2 = \frac{1}{n-k} \sum_{t=1}^n (w_t - \bar{w})^2 \quad \dots \dots \dots (11)$$

Where  $t = k+1, \dots, T$ .  $CUSUM_t$  is the recursive residual and is based on plotting against  $t$ . Under the null hypothesis,  $\beta$  is constant. The CUSUM has zero mean, and variance is proportional to  $t-k-1$ . If the null hypothesis is rejected, then the recursive residual crosses the boundary for some  $t$ . However, the CUSUM test aims to detect precise movements of coefficients.

The tendency of a disproportionate number of recursive residual to have the same sign, it indicates that the coefficients are not constant, and the recursive residuals cross the boundary, as stated by Baltagi (2011). The cumulative sum of the squared test was used in the squared recursive residual and which is based on the plotting against  $t$ . Under the null hypothesis  $\beta = (n-k)/(T-k)$  which varies from 0 to 1. 0 for  $n=k$  and 1 for  $n=T$ . If the null hypothesis is rejected, then the squared recursive residual crosses the boundary and determines the level of the test by Maddala (1998); Zivot (2003) and Baltagi (2011). Ploberger, Kramer, and Alt (1988)

extended the CUSUM test with lagged dependent variables. Ploberger, Kramer, and Alt (1989) had implied the local power of the CUSUM test against heteroskedasticity. For the power problem, they proposed a fluctuation test (in relation with successive parameter estimated rather than recursive residuals). It was first suggested by Ploberger in 1983. Ploberger and Kramer (1990; 1992) extended the CUSUM and CUSUM squared test in the linear regression model with lagged dependent variables and local power of CUSUM. They adopted a dynamic linear regression model to show the structural shift, and also they proposed the fluctuation test rather than recursive residual on parameter estimation. However, a structural shift occurred lately in their sample. They proved that there was a drawback in the CUSUM test, which obtained an asymptotically negative coefficient of its regression. But it does not induce heteroskedasticity of its disturbances rather than on constant coefficients. Subsequently, the CUSUM squared test showed asymptotically identical. Westlund and Tornkvist (1989) experimented with CUSUM, and CUSUM squared test. They were interested in testing the structural stability, for that they employed test statistics and Monte Carlo technique. Parameters estimation of the test statistics varied differently, and Monte Carlo technique experienced the minimal possibility of generalization. Overall, CUSUM and CUSUM squared test statistics was not known for varying parameters.

Another class of tests is Andrews (1993) Sup F test. He considered a test for parameter instability and one-time structural change with unknown change point. This study has nontrivial asymptotic local power against all alternatives for which the parameters are non-constant or where the change point (structural break) which were unknown. However, the structural breaks were known; one can form specifically Walt, Lagrange Multiplier models with no deterministic or stochastic trends. Also, to that Andrews proposed Likelihood ratio-like test based on the Generalised Method of Moments (GMM) estimators with nonlinear models. Also, he provided an asymptotic critical value that is Sup F test Maddala (1998). Here the standard method employed by Andrews.



$\sup_{\pi \in \Pi} W_T(\pi)$ ,  $\sup_{\pi \in \Pi} LM_T(\pi)$ , and  $\sup_{\pi \in \Pi} LR_T(\pi)$  ..... 12

Where  $\Pi$  is a pre-specified subset of  $(0,1)$ ? The test of the form divided into three categories; those are:  $\sup_{\pi \in \Pi} LRT(\pi)$  is the Likelihood ratio test. In this case test statistics parameter unspecified  $\pi$  with parameter space  $\Pi$ . However,  $\sup_{\pi \in \Pi} WT(\pi)$ , and  $\sup_{\pi \in \Pi} LMT(\pi)$  tests are asymptotically equivalent to  $\sup_{\pi \in \Pi} LRT(\pi)$  under the null and local alternatives under suitable assumptions. The test statistics  $\sup_{\pi \in \Pi} WT(\pi)$ .....  $\sup_{\pi \in \Pi} LRT(\pi)$  has derived from Roy's type one principle. Andrews test statistics proved that there is weak asymptotic optimality propriety against local alternatives for the large sample size and small significance level. Subsequently, this study certainly concentrated an arbitrarily weighted function of  $g(\{WT:\pi \in \Pi\})$ ,  $g(\{LMT:\pi \in \Pi\})$  and  $g(\{LRT:\pi \in \Pi\})$ . Also, to this, the test statistics was found in some advantage in terms of weighted average power for specific weighted function over test statistics of the sup form.

Hypothesis on this study; where that, the unknown change point is occurred (based on the political or institutional change that has occurred in a known time period) on restricted interval  $\Pi_c(0, 1)$  and where the timing of structural changes occurred (that is an exogenous event)  $(0, 1)$ . The general diagnostic test employed here restricted alternatives  $U_{\pi \in \Pi} H1T(\pi)$ , where  $H1:\beta_s \neq \beta_t$  for some  $s, t \geq 1$ . Andrews (1993) analysis was applied to nonlinear models as he provided better power properties with Sup F test (asymptotic critical values for 1, 2.5, 5 and 10 percent of significance levels) compared with the fluctuation test and the CUSUM test. Subsequently, Andrews and Ploberger (1994) applied the optimal analysis, when a nuisance parameter was present under the alternative. For this purpose, they considered stationary series strictly, and a weighted average power criterion was used to generate an optimal test. However, standard LM, Walt, and LR test have introduced. Eventually, new optimal analysis employed on the LR and Walt test and LR test have not found to be an optimal test.

### Multiple Unknown Structural Breaks

In recent decades, identification of multiple strange breaks has drawn attention. Bai (1997), Bai

and Perron (1994, 1997, 1998, 2003, 2003a and 2006) and Perron (2005) have contributed significantly to the literature.

To estimate multiple breaks, there are various alternative procedures under the Bai-Perron class of tests. These procedures are global maximizer, sequential analysis, and hybrid versions with both components.

The sequence of discussion on Bai and Perron; models allow us for general forms of serial correlation and heteroscedasticity in the errors, lagged dependent variables, trending regressors, as well as different distributions for the mistakes and the regressors across segments. Subsequently, a partial structural change model was also employed; it does not all parameters are subject to shifts.

Bai and Perron (1994), empirically examined inference models with a structural break that was simultaneous rather than sequential methods (successive estimation of each breakpoint). The target was to identify the determinants of several breaks estimated the number of the breakpoint at given breaks and result estimators. They described that the partial structural change model where all parameters were constant. The aim was to identify the successful method of estimated each breakpoint rather than the location of breaks. They have concentrated on no structural changes versus an arbitrary number of changes and their estimated null hypothesis (l) verses (l+1) changes, for the analytical purpose they used Sup Walt test. It includes the linear regression model with multiple structural breaks by minimizing the sum of squared residuals. The presence of structural break is being determined by the properties of the estimators, number of breaks and estimated break dates, etc.,

$$\begin{aligned} y_t &= x_t' \beta + z_t' \delta_1 + u_t \dots \dots \dots 13 \quad t = 1, 2, \dots \dots T1 \\ y_t &= x_t' \beta + z_t' \delta_2 + u_t \quad t = T1 + 1, \dots \dots T2 \\ &\vdots \\ &\vdots \\ &\vdots \\ y_t &= x_t' \beta + z_t' \delta_{m+1} + u_t \quad t = t_m + 1, \dots \dots T \end{aligned}$$

Where  $y_t$  is a dependent variable at time  $t$ , vector of covariates  $x_t$  ( $p \times 1$ ) and  $z_t$  ( $q \times 1$ ) and vector of coefficients  $\beta$  and  $\delta_j$  ( $j = 1, \dots, m+1$ ) and  $u_t$  error

term at time  $t$ . They treated breakpoints as explicitly unknown that is  $T_1, T_2, \dots, T_m$ .  $T$  is observed as unknown regression coefficient  $y_t, x_t, z_t$ . However, the parameter  $\beta$  was not subjected to shift where the coefficient ( $p=0$ ). Therefore, all the coefficients are subjected to change in a pure structural change model and for more details, see Bai and Perron (1994). They have used US ex-post real interest rate quarterly data from 1961:1 to 1986:3. For empirical verification, they allowed up to 5 segments and identified two breaks dates (1972:3 and 1980:3) estimation under global minimization. However, it is useful for the treatment of linear regression models with multiple structural breaks. They did not allow a convergence rate of sequential estimators but estimated the convergence rate of breakpoints. This study follows the approach of limiting distribution of break dates (for global minimization behavior and social behavior) Bai and Perron (1995b) and sequential estimation of multiple breaks, Bai and Perron (1997). Finally, Justin Bai, the dissent that when parameter  $\beta$  was not subjected to shift then this study does not allow  $T$ -consistent  $y_t, x_t$  and  $z_t$  convergence rate of sequential estimators or breakpoints and for details of sequential estimation of multiple breaks see Bai and Perron (1997). To estimate multiple breaks, there are various procedure to follow. Those procedures are discussed next.

First is to employ an appropriate model based on Bai and Perron continuous argument. For that Bai and Perron (1998) employed multiple linear regression with  $m$  breaks and  $m+1$  regime

$$y_t = x_t' \beta + z_t' \delta_j + u_t \dots \dots \dots 14$$

Where  $y_t$  is the independent variable.  $x_t$  and  $z_t$  is the vector of covariates;  $x_t$  ( $p \times 1$ ) and  $z_t$  ( $q \times 1$ ). ( $t = T_j - 1 + 1 \dots T_j$  ( $j = 1, \dots, m+1$   $T_0 = 0$  and  $T_{m+1} = T$  used for the convention). The unknown breakpoints were  $T_1, \dots, T_m$ .  $\beta$  and  $\delta$  is the vector of coefficients that is ( $j = 1, \dots, m+1$ ) and error term  $u_t$ . The regression coefficients are ( $y_t, x_t$ , and  $z_t$ ) with  $T$  observations.  $\beta$  is constant because they used partial structural change model. When all the coefficient are subjected to change, then it could be a pure structural change model with  $p=0$ .

The above model has expressed in the matrix form

$$Y = X \beta_0 + Z \delta + U \dots \dots \dots 15$$

Where,  $y_t$  is ( $y_1, \dots, y_T$ ),  $X$  is ( $x_1, \dots, x_T$ ),  $U$  is ( $u_1, \dots, u_T$ ),  $\delta$  is ( $\delta_1, \delta_2, \dots, \delta_{m+1}$ ) and  $Z$  is at  $m$ -partition ( $T_1, \dots, T_m$ ) and its diagonal matrix is  $Z$  that is  $Z_1, \dots, Z_{m+1}$  with  $Z_i$  is equal to  $z_{T_i-1+1}, \dots, z_{T_i}$ . They treated 0 as superscript, which denotes the true value of the parameters.

To estimate the unknown regression coefficient ( $\beta_0, \delta_0, \dots, \delta_{m+1}, T_0, \dots, T_m$  assumed that  $\delta_0 \neq \delta_{0i} \neq 1$  ( $1 \leq k \leq m$ ). an unknown number of breaks are treated for the real value of  $m$  discrete shift.

For the least-squares principle-minimizing sum of squared residuals  $S_T = \sum_{t=T_i-1+1}^{T_i} (y_t - x_t' \beta - z_t' \delta_i)^2$  which are obtained as the coefficient  $\beta$  and  $\delta_j$  and  $T_1, \dots, T_m$  which denotes  $\{T_j\}$  for each  $m$ -partitions.  $S_T (T_1, \dots, T_m)$  is the resulting sum of squared residuals and the estimated breakpoints  $T_1, \dots, T_m$

$$(\hat{T}_1, \dots, \hat{T}_m) = \arg \min_{T_1, \dots, T_m} S_T (T_1, \dots, T_m) \dots \dots \dots 16$$

$m$ -Partition ( $T_1, \dots, T_m$ ) which overall minimization to  $T_i - T_{i-1} \geq q$ . The first issue in the Bai-Perron test is to find all possible breakpoints and acceptable segments. Dynamic programming is used in obtaining breakpoints. It is considered by Bai and Perron (1998 and 2003). Table 3.1 and 3.2 present triangular matrices of sums of squared residuals with  $T=25$ ,  $h=5$ , and  $m=2$ . These Tables show the possible breakpoints and a calculation of acceptable segments, respectively.

Bai and Perron (1998 and 2006) have shown two tests of the null hypothesis, that is, no structural difference against an unknown number of breaks given some upper bound  $M$ . The first is considered double maximum checks. There are two subsets within this: an equal-weight version (UDMax) and the second test finds individual weights which give equal marginal  $p$ -values across  $m$  (WDMax). The detailed account is given here.

**Figure 1 The Triangular Matrix of Sums of Squared Residuals Terminal Date**

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
1	xa	xa	xa	xa	*	*	*	*	*	*	*	*	*	*	*	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
2		xa	xa	xa	xa	xc	xc	xc	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
3			xa	xa	xa	xc	xc	xc	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
4				xa	xa	xc	xc	xc	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
5					xa	xc	xc	xc	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
6						xa	xc	xc	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
7							xa	xc	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
8								xa	xc	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
9									xa	xc	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
10										xa	xc	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
11											xa	xc	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
12												xa	xc	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
13													xa	xc	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
14														xa	xc	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
15															xa	xb	xb	xb	xb	xb	xb	xb	xb	xb	xb
16																xa	xb	xb	xb	xb	xb	xb	xb	xb	xb
17																	xa	xb	xb	xb	xb	xb	xb	xb	xb
18																		xa	xb	xb	xb	xb	xb	xb	xb
19																			xa	xb	xb	xb	xb	xb	xb
20																				xa	xb	xb	xb	xb	xb
21																					xa	xb	xb	xb	xb
22																						xa	xb	xb	xb
23																							xa	xb	xb
24																								xa	xb
25																									xa

**Source:** Bai and Perron (2003)

Notes: The vertical number indicates the initial date of a segment while the horizontal number indicates the terminal data. For example, the entry (4, 10) shows a section that starts on year 4 and ends at date 10, hence having seven observations.

- xa indicates a segment not considered since it must be at least of length 5.

- xb shows a section not found since otherwise there would be no place for 3 parts of period 5.
- xc indicates a portion not considered since otherwise there would be no place for a section of range 5 prior it.
- A\* indicates an admissible segment

**Table 1 Explanation of Global Minimisation**

Condition	Specification	Example	
Total unrestricted segment	$T = 25, m = 2$ (segment) and $h = 5$ length (distance) $T(T+1)/2$	Total = $25 \times 26 / 2 = 325$	$T = 53, h = 8, m = 5$ Total = $53 \times 54 / 2 = 1431$
Every segment at least at length $h$ where ( $h \geq q$ )	$= (h-1)T - (h-2)(h-1)/2$	$xa = (4) 25 - (3)(4)/2$ $xa = 100 - 6 = 94$	$xa = (7) 53 - (6)(7)/2$ $xa = 371 - 42/2$ $xa = 350$



Largest segment should be short enough to allow other segments. At a segment starts between 1 to h than the length of this segment is T-hm, more breaks allowed.	$=h2m(m+1)/2$	$xb = 52*2(3)/2 = 25*3$ $xb = 75$	$xb = 82*5(6)/2 = 64*15$ $xb = 960$
Segments not considered	$T(h-1)-mh(h-1)-(h-1)2-h(h-1)/2$	$xc = 25(4)-10(4)-(16)-20/2$ $xc = 25(4)-10(4)-(16)-20/2$ $xc = 100-40-16-10=34$	$xc = 53(7)-5*8(7)-(7)2-8(7)/2$ $xc = 371-270-49-28$ $xc = 24$
Total acceptable segments		122	117

In general trimming (h) is not necessary to fix to q. However, trimming has chosen independently based on the number of regressors exists. There are two instances pertain to the nature of error distribution about the regressors based on the assumption. First is when the regressors contain no lagged values, then the residuals (error term ut) permits substantial correlation and heteroskedasticity. Second is when the regressors contain lagged values; then the residuals permit no serial correlation. Subsequently, those two instances allow different distributions for regressors and the error distribution across segments (Bai and Perron 2006).

For constructing a confidence interval, Bai-Perron strategy was to adopt the asymptotic framework where the size/magnitude of the shifts converge to zero leads to a sample size increase and break dates based on asymptotic distribution. For the coefficient covariance, Bai and Perron used HAC estimator, which is first-order autoregressive approximation with Quadratic Spectral Kernel, Andrews bandwidth [each element of the vector {ztut} and over segment/vector {ztut}].

The second test, WDMax, has marginal p-values which are equal across m values. They have implied weight on q, and then significance level is on a, then the test precisely followed  $c(q, a, m)$ . However,  $\sup(\lambda_1, \dots, \lambda_m) \in \Lambda \in FT(\lambda_1, \dots, \lambda_m; q)$  is the asymptotic critical value. Subsequently, when a1 is equal to 1 and form is greater than 1 as am is equal to  $c(q, a, 1)/c(q, a, m)$  are the weights then it is denoted by

$$WDmax F_T(M, q) = \max_{1 \leq m \leq M} \frac{c(q, a, 1)}{c(q, a, m)} F_T(\lambda_1, \dots, \lambda_m; q)$$

The asymptotic equal version is

$$UDmax F_T^+(M, q) = \max_{1 \leq m \leq M} \frac{c(q, a, 1)}{c(q, a, m)} \sup_{\lambda_1, \dots, \lambda_m \in \Lambda} F_T^+(\lambda_1, \dots, \lambda_m; q)$$

However, the significance level of WD max FT (M, q) have chosen weights themselves based on a, unlike UD max FT (M, q) and based on Bai and Perron the critical values and the trimming is equal to .05, .10 and .15 then 5 breaks are allowed. If the trimming is equal to .20, then 3 breaks are allowed, and the trimming is equal to .25, then the break is 2.

Apart from the double maximum tests, information criteria such as BIC and LWZ were used, for identifying the maximum number of breaks. In the presence of serial correlation, another information criterion– AIC does not perform well. BIC does not perform very well in the presence of a lagged dependent variable. LWZ performs better under the null hypothesis of no break but underestimates the number of breaks when some are presents.

Another class of test is the sequential tests of L versus L+1 breakpoints. It is primarily focused on the difference between the sum of squared residuals obtained by L versus and L+1. This test applied each observations Ti-1 to T1(i=1,.....L+1) and each segment as well. This test is not necessary to use global sum of squared residuals to Tbut it has focused on break fractions that is  $\lambda_i = T_i/T$ , and it has converged actual value of T. Break dates selected based on the overall minimum amount, that is where the model L+1 shows total minimum value of the sum of squared and it is sufficiently smaller than the model L. Bai and Perron (2006, p 19–20) has recommended the use of sequential L versus L+1 approach for tracing the breakpoints. The exact

recommendation is as follows:

- The adequate size of the test has to be specified under the hypothesis of no structural break. Any value of the trimming is allowed in the regression model with sufficient size when the tests are not allowed such as serial correlation and heterogeneity or error across segments (and not presented in the dataset itself). In such cases, serial correlation and heterogeneity or errors across segment features are present then the higher trimming procedure is needed. Example, when heterogeneity is present in error or the dataset (sample size=120), then the trimming is 0.15. If a serial correlation is allowed, then the edging can be 0.20. If the sample size is vast, then the above trimming procedure can be reduced (pg.19).
- When breakpoints are less and serial correlation is allowed, then the breakpoint selection is made by using BIC, which would work efficiently less null hypothesis. Under the null hypothesis, LWZ criterion works well with a higher penalty (even when the serial correlation is present). When we impose a higher penalty along with breaks than we observe lousy performance in it, however, while using the model selection method of information criterion, we cannot consider the presence of heterogeneity across segments. Overall, the break selection works best in the sequential process.
- Their opinion is to improve sequential procedure with the help of UDMax or WDMax for breaks. That is if a break date is present in the UDMax or WDMax test. The formula used for this is  $\text{Sup } F(L+1 \text{ versus } L)$  statistic procedure using global minimization of the SSR.
- For the practical justification, the power of the UDMax or WDMax test is almost equal to the power of an analysis of no structural changes versus an alternative number of structural reforms.
- When the break dates are too small or too big, it is an encouraging result. When the valid break values are missing or misleading, then the confidence intervals are inadequate. Whereas, when a more significant amount of breakpoint is observed, then there will be a real value of

confidence interval, which will be accurate while estimating.

- The power of the test and selecting the breakpoint accuracy has improved with the help of correcting serial correlation, heterogeneity in the distribution and error across the segment.

Bai and Perron (2003 & 2006) adopted various assumptions for autocorrelation along with error distributions and heteroskedasticity in the regressors. In practice, the HAC technique provided various constraints to fix the error distribution and regressors across segments in the general framework. In general, Bai and Perron adopted different assumptions for autocorrelation along with error distributions and heteroskedasticity in the regressors.

- If the error distributions are allowed to correlate, then they denote  $\text{cor}_u = 1$  and where there is no serial correlation which will be indicated as  $\text{cor}_u = 0$ .
- If heterogeneous distributions presented across segments in the regressors, then they denote  $\text{het}_z = 1$  and where homogenous distributions performed across a section in the regressors which will be indicated as  $\text{het}_z = 0$ .
- Let heterogenous ( $\text{het}_u = 1$ ) residual variance permits across segments and where ( $\text{het}_u = 0$ ) presented the same variance in this case there are restriction has been shown, that is a) identical distribution of regressors across segments, b) equal distribution of errors across sectors, c) identical distribution of errors and data across divisions, d) where serially uncorrelated errors presents, e) serially uncorrelated errors and identically distribution across segments, f) serially uncorrelated errors and identical distribution of regressors across sectors, g) serially uncorrelated errors and equal distribution of errors and data across segments. Various constructions for the different specification of the estimates about the limiting distribution and the confidence interval see Bai and Perron (1998, 2003 and 2006). Based on the above assumptions, they might not stand true for the precise estimation when a valid restriction is imposed.

Taking into consideration, cases from Bai and Perron (1998); example one is no serial correlation and same distribution for the errors across segments

(cor\_u=0, het\_u=0); case two is no serial correlation in the errors and different variance for the residuals across sectors (cor\_u=0, het\_u=1). Imposing typical distribution (het\_z=0) regressors across segments brought the tests with worst properties when the data has invariant distribution. They provided a relevant asymptotic critical value of the multiple breaks and the sequential (L+1 | L) for trimming cost, which is equal to .05. When k is equal to 1 to 9 and q is equal to 1 to 10. In this case of no serial correlation in the errors and different variance for the residuals across segments (cor\_u = 0, het\_z=1, het\_u = 0) in which small trimming is allowed arbitrarily. Mainly, where 8 is considered to be the maximum number of a break then the trimming is equal to .10. Similarly, when trimming is equivalent to 0.15 then the maximum number of breaks allowed is 5, when cutting is equal to .20 then 3 breaks allowed and when decorating is equal to .25, then two breaks permitted.

The above consideration has required that the specification of a particular number of breaks (m), against the alternative hypothesis. To introduce inference, a pre-specified number of breaks was not encouraged often by the researchers. To the extent, Bai and Perron (1998 and 2006) have shown two tests of the null hypothesis. That is no structural break against an unknown number of breaks given some upper bound M. The first is considered double maximum tests to fix an equal weight and the second test is considered individuals weight test which is similar and applied marginal p-values across m.

Double maximum Test:  $D_{max} FT(M, q, a, 1, \dots, aM) = \max_{1 \leq m \leq M} \sup_{\lambda_1, \dots, \lambda_m \in \Lambda \in FT^*(\lambda_1, \dots, \lambda_m; q)}$  fixed weights is  $\{a_1, \dots, a_m\}$  and then all weights equal to unity. The modified version of  $UD_{max} FT(M, q) = \max_{1 \leq m \leq M} \sup_{(\lambda_1, \dots, \lambda_m) \in \Lambda \in FT(\lambda_1, \dots, \lambda_m; q)}$ . M is fixed, the sum of m is  $F(\lambda_1, \dots, \lambda_m; q)$  depends on chi-squared random variables with q degrees of freedom, each one divided by m.

Equal weighted version:  $UD_{max} F^*T(M, q) = \max_{1 \leq m \leq M} \sup_{(\lambda_1, \dots, \lambda_m) \in \Lambda \in F^*T(\lambda_1, \dots, \lambda_m; q)}$ ; Bai and Perron used asymptotical equal version is  $UD_{max} FT(M, q) = \max_{1 \leq m \leq M} F_T(\lambda_1, \dots, \lambda_m; q)$ . Where  $\lambda_j = Tj/T = 1, \dots, m$  obtained estimates the global minimization of the sum of squared residuals of the breakpoints.

The second test, marginal p-values which are equal across m values. They have implied weight on q, and then significance level is on a, then the test precisely followed  $c(q, a, m)$ . However,  $\sup_{(\lambda_1, \dots, \lambda_m) \in \Lambda \in FT(\lambda_1, \dots, \lambda_m; q)}$  is the asymptotic critical value. Subsequently, when a is equal to 1 and for m is greater than 1 as am is equal to  $c(q, a, 1)/c(q, a, m)$  are the weights then it is denoted by

$$WD_{max} F_T(M, q) = \max_{1 \leq m \leq M} \frac{c(q, a, 1)}{c(q, a, m)} F_T(\lambda_1, \dots, \lambda_m; q)$$

The asymptotic equal version is

$$UD_{max} F_T^*(M, q) = \max_{1 \leq m \leq M} \frac{c(q, a, 1)}{c(q, a, m)} \sup_{\lambda_1, \dots, \lambda_m \in \Lambda} F_T^*(\lambda_1, \dots, \lambda_m; q)$$

However, the significance level of  $WD_{max} FT(M, q)$  have chosen weights themselves based on a, unlike  $UD_{max} FT(M, q)$  and based on Bai and Perron the critical values and the trimming is equal to .05, .10 and .15 then 5 breaks are allowed. If the trimming is equal to .20, then 3 breaks are allowed, and the trimming is equal to .25, then the break is 2.

Another class of test is L versus L+1 breakpoints. It is primarily focused on the difference between the sum of squared residuals obtained by L versus and L+1. This test applied each observations  $T_{i-1}$  to  $T_i (i=1, \dots, L+1)$  and each segment as well. This test is not necessary to use global sum of squared residuals to T but it has focused on break fractions that is  $\lambda_i = T_i/T$ , and it has converged actual value of T. Break dates selected based on the overall minimum amount, that is where the model L+1 shows total minimum value of the sum of squared and it is sufficiently smaller than the model L.

### The Repartition Procedure

This technique re-estimates each of the breakpoints based on the initial estimates (initial T-consistent estimator  $k_{i0}$  ( $i=1, 2$ ) were obtained). To estimate  $k_{10}$  the subsample  $[1, k_2]$  were used and to determine  $k_{20}$  the subsample  $[k_1, T]$  were used, the resulting estimators by  $k_1^*$  and  $k_2^*$  respectively. The proximity of  $k_i$  to  $k_{i0}$  and effectively used sample  $[k_{i-10}+1, k_{i+10}]$  to estimate  $k_{i0}$  ( $i=1, 2$  with  $k_{00}=1, k_{30}=T$  (Bai and Perron 1997).

## Sequential Procedure

The number of breaks is unknown  $m$  when the first breakpoint is observed, and the whole sample is divided into two subsamples. It is consisting of the first observation and the second sample consisting of the rest of the view. This procedure would continue until the test fail to reject the null hypothesis of no structural break (Bai and Perron 1998).

There is a standard procedure considered to follow selecting model, which is an Informative Bayesian Criterion (BIC) suggest by Yao (1988). He proposed that the consistent estimation of several breaks, which is a sequence of random variables with shifts in the mean. Similarly, an alternative method showed by Liu, Wu, and Zidek (1997), which is a Schwarz' criterion. This procedure suggested either start with a small number of breaks or started with no tears. This procedure is similar to the sequential estimation of a structural break, for more details see Bai and Perron (1998 and 2006).

## Bai-Perron tests have Revealed the Following Limitations

First, the time-series economic data has more than one structural break many times; Second, while Bai-Perron class of tests support the determination of multiple breaks but the number and location of breaks are highly susceptible to the kind of Bai-Perron test employed as well as the assumptions regarding number of breaks in case of tests based on known breaks and trimming parameters as suggested by the sensitivity analyses. Finally, a minor change in the length of the series or choice of real price series instead of a nominal one also affects the outcomes. Hence, the role of sensitivity analysis becomes essential in such a scenario.

## Discussion

The conventional test for structural break has been the well-known chow test (Chow, 1960). In its original form, the chow test involves estimation of regression models for the two segments before and after the suspected breakpoint. For checking if the two parts are statistically different in terms of the estimated model, one has to employ the F-test on the Residual Sum of Squares (RSS) for the restricted (Full model) and unrestricted model (sum of RSS in

two segments). If the estimated F-statistics is higher than the critical value, it suggests rejection of the null of no breakpoint. A dummy variable version of chow test is more popular. In this case, a dummy is introduced in the model to see if the intercept of pre and post breakpoint of this estimated equations are different. An interaction dummy is also included to check whether the slopes of the equations are statistically different.

Advancement in econometrics in the analysis of structural break(s) in time series has been phenomenal. Around the time of chow test, the work of Quandt (Quandt, 1960) revealed that instead of testing for a known break due to a-priori reasoning based on some historical event, it is advisable to look for the unknown break which yields the optimal value for a test similar to Chow test. It is identical to a max-chow test for all possible breaks within a time segment. Quandt (1960) and subsequently, Andrews (1993) have developed a test known as Quandt-Andrews test, which is a likelihood ratio test to check for the unknown break. This has the advantage of taking care of nonlinearity of models in contrast to ordinary least squares.

A number of other contributions have been significant in the field of structural break before the most recent test -Bai-Perron was introduced (Farley and Hinich, 1970; Brown et al., 1975; Farely et al., 1975; Kim and Siegmund, 1989; Perron, 1989; 1991; 1998; 2005; Andrews, 1993). However, none of these tests could present a theoretically comprehensive model for tracing "multiple" unknown breaks. The recent works of Bai and Perron (1994, 1998, 2003, 2003a), known as Bai-Perron tests, have been significant in clarifying the econometric issues involved in various ways of looking for multiple structural breaks in time series data. Despite these theoretical developments, the practical applications of Bai-Perron tests were limited to Bai-Perron (2003), and Zeileis et al. (2003) clarified the computational issues involved in applying these tests. Within the Bai-Perron class of tests, depending on the procedure adopted (maximum double trials,  $L$  Vs.  $L+1$ , known Vs. unknown breaks, etc.) and the assumption regarding the size of the segments/trimming parameters and number of tears, the results could differ.

Some issues are raised about Bai-Perron Tests. There are difficulties in making finite conclusions about the exact number and position of multiple breaks as they are susceptible to assumptions made about the number of breakpoints, size of segments/trimming parameters. Dholakia and Sapre (2011) found different number and position of breakpoints when assumptions about several breakpoints, size of parts, the base year of the output data and the size of the reference period were changed. Given the problem of the inexactness of break positions and their number as in the case, one has to test Bai-Perron and the Quandt-Andrews breakpoint, even though it renders a single breakpoint.

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