Convex Fuzzy Soft Metric Space

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Abstract
In this paper, we investigate the concept of fuzzy soft metric space in terms of fuzzy soft points. The convex structure of fuzzy soft metric spaces is defined and we introduce the convex fuzzy soft metric space. Also we established the fixed point theorem of convex fuzzy soft metric space.

Keywords: Fuzzy soft metric space, Convex structure, Convex Fuzzy soft metric space, Fixed point

1. Introduction

Many complicated problems in economics, engineering, medical sciences and many other fields involve uncertain data. These problems in life, cannot be solved using classical mathematics. There are two types of mathematical tools to deal with uncertainties namely fuzzy set theory introduced by L.A. Zadeh and the theory of soft sets initiated by Molodstov which helps to solve problems in all areas. In this paper we define fuzzy soft metric space and the convex structure of fuzzy soft metric space. In section 2, we review the necessary definitions and preliminary notions of fuzzy soft metric space used throughout this paper. In section 3, we introduce the concept of convex fuzzy soft metric space and its characterizations and also established the fixed point theorem of convex fuzzy soft metric space.

2. Preliminaries

Definition 2.1

Let X be an initial universe and E be a set of parameters and A ⊆ E. Let P(X) denotes the power set of X. A pair (F, A) is called a soft set over X if F is a mapping from A into the set of all subsets of X. That is, F: A → P(X).

Definition 2.2

Let X be an initial universe set and E be the set of parameters and A ⊆ E. Then the pair (F, A) is called fuzzy soft set on X if F is a mapping defined by F: A → P(X), where P(X) denotes the collection of all fuzzy subsets of X. It is denoted by F_A and the set of all fuzzy soft set is denoted by FS(X, E).

Definition 2.3

A fuzzy soft set F_A over X is said to be fuzzy soft point if there is exactly one e ∈ A, such that F(e)≠ Φ and F(e') = Φ for all e'∈ A\{e}. It is denoted by F_e.

Definition 2.4

Let F_E ∈ FS(X, E). If $F_E(e) = 1$, for all e ∈ E, then F_E is called absolute fuzzy soft set.

Definition 2.5

Let F_E be the absolute fuzzy soft set and (A)* denote the set of all non negative fuzzy soft real numbers and the collection of all fuzzy soft points of a fuzzy soft set F_E be denoted by FSC(F_E). A mapping d: FSC(F_E) × FSC(F_E) → (A)* is said to be a fuzzy soft metric on F_E if it satisfies the following conditions:

(FSM_1) d(F_{e_1}, F_{e_2}) ≥ 0, for all F_{e_1}, F_{e_2}∈ FSC(F_E).

(FSM_2) d(F_{e_1}, F_{e_2}) = 0 if and only if $F_{e_1} = F_{e_2}$.

(FSM_3) d(F_{e_1}, F_{e_2}) = d(F_{e_2}, F_{e_1}), for all F_{e_1}, F_{e_2}∈ FSC(F_E).

(FSM_4) d(F_{e_1}, F_{e_3}) ≤ d(F_{e_1}, F_{e_2}) + d(F_{e_2}, F_{e_3}), for all F_{e_1}, F_{e_2}, F_{e_3}∈ FSC(F_E).

The fuzzy soft set F_E with the fuzzy soft metric d is called the fuzzy soft metric space and it is denoted by (F_E, d).

Definition 2.6

The fuzzy soft set F_A is called fuzzy soft open set if for every $G_e \in F_A$ there exists $\tilde{\tilde{r}} > 0$ such that $\tilde{\tilde{B}}(G_e, \tilde{\tilde{r}}) \subseteq F_A$ where $\tilde{\tilde{B}}(G_e, \tilde{\tilde{r}}) = \{H_e \in F_A : d(H_e, G_e) < \tilde{\tilde{r}}\}$ and $\tilde{\tilde{B}}(G_e, \tilde{\tilde{r}})$ is called an open sphere or open ball with center G_e and radius $\tilde{\tilde{r}}$.

Definition 2.7

Let (F_E, d) be a fuzzy soft metric space. A soft set $F_A \subset F_E$ is said to be fuzzy soft closed in F_E with respect to d if its complement ${F_A}^c$ is fuzzy soft open in (F_E, d).

Definition 2.8

The fuzzy soft set F_E is said to be convex fuzzy soft set if for all F_{e_1}, F_{e_2}∈ F_E, F(λF_{e_1}+(1-λ) F_{e_2}) ⊇ F(F_{e_1})∩F(F_{e_2}), where λ∈[0, 1].

Definition 2.9

Let (X, d) be a metric space and I=[0,1]. A mapping W: X × X × I → X is said to be a convex structure on X if for each (x, y, λ) ∈ X × X × I and u∈ X, Then

$$d(u, W(x, y, \lambda)) \leq \lambda d(u, x) + (1-\lambda) d(u, y).$$

The metric space (X, d) together with a convex structure W is called a convex metric space, which is denoted by (X, d, W).

Definition 2.10

Let (X, d) be a metric space. The mapping T: X → X is fuzzy contractive if there exists k∈ (0, 1) such that d(Tx, Ty, t) ≤ kd(x,y,t) where x,y ∈ X and t > 0.

Definition 2.11

Let (X, d) be a metric space and the mapping T: X → X is fuzzy contractive mapping. Then the sequence $\{x_n\}$ is said to be contractive sequence if d(x_n, x_{n+1}, t) ≤ kd(x_0, x_1, t) where x, y∈ X, n∈ N and 0 < k < 1.

3. Convex Fuzzy Soft Metric Space

Definition 3.1

Let (F_E, d) be a fuzzy soft metric space and I= [0, 1]. Let FSC(F_E) be the collection of all fuzzy soft points. A mapping V: FSC(F_E)× FSC(F_E)×I → FSC(F_E) is said to be convex structure on F_E if for each (F_{e_1},F_{e_2},λ) ∈ FSC(F_E)× FSC(F_E)×I, $F_e \in F_E$ and λ∈[0, 1],

$$d(F_e, V(F_{e_1}, F_{e_2}, \lambda)) \leq \lambda d(F_e, F_{e_1}) + (1-\lambda) d(F_e, F_{e_2})$$

A fuzzy soft metric space (F_E, d) together with a convex structure V is called a convex fuzzy soft metric space, which is denoted by (F_E, d, V).

Definition 3.2

Let (F_E, d, V) be a convex fuzzy soft metric space. A non empty subset $F_A$of F_E is said to convex if V(F_{e_1}, F_{e_2}, λ) ∈ F_A whenever (F_{e_1}, F_{e_2}, λ) ∈ FSC(F_A)× FSC(F_A)×I

Definition 3.3

Let (F_E, d, V) be a convex fuzzy soft metric space and F_A be a convex subset of X. A mapping f on F_A is said to be self mapping if

f(V(F_{e_1}, F_{e_2},λ))= V($f(F_{e_1})$, $f(F_{e_2})$, λ) for all F_{e_1}, F_{e_2}∈ F_A and λ ∈ I.

Definition 3.4

Let: F_E→ F_E be a mapping on F_E. A point F_e∈ F_E is called a fixed point of f if $f(F_e)$ = F_e.

Theorem 3.5

Let (F_E, d, V) be a convex fuzzy soft metric space. Then the following conditions are hold: For all (F_{e_1}, F_{e_2}, λ) ∈ FSC(F_E)× FSC(F_E)×I

i. d(F_{e_1}, F_{e_2}) = d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ))
ii. d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) = (1-λ) d(F_{e_1}, F_{e_2})
iii. d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ)) = λ d(F_{e_1}, F_{e_2})
iv. d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = d(F_{e_2}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = $\frac{1}{2}$ d(F_{e_1}, F_{e_2})

Proof

(i) For any (F_{e_1}, F_{e_2},) ∈ FSC(F_E)× FSC(F_E)×I

To prove d(F_{e_1}, F_{e_2}) = d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ))

Suppose d(F_{e_1}, F_{e_2}) < d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ))

≤ λ d(F_{e_1}, F_{e_1})+(1-λ) d(F_{e_1}, F_{e_2})+λ d(F_{e_1}, F_{e_2})+(1-λ) d(F_{e_2}, F_{e_2})

= (1-λ) d(F_{e_1}, F_{e_2}) + λ d(F_{e_1}, F_{e_2})

= (1-λ +λ) d(F_{e_1}, F_{e_2})

Hence d (F_{e_1}, F_{e_2}) < d(F_{e_1}, F_{e_2}), which is impossible.

Hence d(F_{e_1}, F_{e_2}) = d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) + d (F_{e_2}, V(F_{e_1}, F_{e_2}, λ)).

(ii) To prove d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) = (1-λ) d(F_{e_1}, F_{e_2})

Now,

d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) ≤ λ d(F_{e_1}, F_{e_1}) + (1- λ) d(F_{e_1}, F_{e_2})

⇒ d (F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) ≤(1- λ) d(F_{e_1}, F_{e_2}) ------ (1)

On the other hand,

(1-λ) d(F_{e_1}, F_{e_2}) = d(F_{e_1}, F_{e_2}) - λ d(F_{e_1}, F_{e_2})

By (i), (1-λ) d(F_{e_1}, F_{e_2}) = d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ)) - λ d(F_{e_1}, F_{e_2})

But d (F_{e_2},V(F_{e_1}, F_{e_2}, λ)) ≤ λ d(F_{e_1}, F_{e_2})

Then (1-λ) d(F_{e_1}, F_{e_2}) ≤ d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ))+ λ d(F_{e_1}, F_{e_2})- λ d(F_{e_1}, F_{e_2})

Hence (1-λ) d(F_{e_1}, F_{e_2}) ≤ d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ))---(2)

Form (1) and (2), we get d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) = (1-λ) d(F_{e_1}, F_{e_2})

(iii) To prove d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ)) = λ d(F_{e_1}, F_{e_2})

By (i) d(F_{e_1}, F_{e_2}) = d(F_{e_1}, V(F_{e_1}, F_{e_2}, λ)) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ))

By (ii) d(F_{e_1}, F_{e_2}) = (1-λ) d(F_{e_1}, F_{e_2}) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ))

= d (F_{e_1}, F_{e_2}) - λ d(F_{e_1}, F_{e_2}) + d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ))

Hence d(F_{e_2}, V(F_{e_1}, F_{e_2}, λ)) = λ d(F_{e_1}, F_{e_2})

(iv) To prove d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = d(F_{e_2}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = $\frac{1}{2}$ d(F_{e_1}, F_{e_2})

By the definition of the convex structure,

d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$))≤s $\frac{1}{2}$ d(F_{e_1}, F_{e_1}) +(1- $\frac{1}{2}$) d(F_{e_1}, F_{e_2})

d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$))≤ $\frac{1}{2}$ d(F_{e_1}, F_{e_2})

By (i), d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) ≤ $\frac{1}{2}$d(F_{e_1}, V(F_{e_1}, F_{e_2}, $\frac{1}{2}$)) + $\frac{1}{2}$d(F_{e_2}, V(F_{e_1}, F_{e_2}, $\frac{1}{2}$))

⇒ d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) - $\frac{1}{2}$d(F_{e_1}, V(F_{e_1}, F_{e_2}, $\frac{1}{2}$)) ≤ $\frac{1}{2}$d(F_{e_2}, V(F_{e_1}, F_{e_2}, $\frac{1}{2}$))

⇒ $\frac{1}{2}$d(F_{e_1}, V(F_{e_1}, F_{e_2}, $\frac{1}{2}$)) ≤ $\frac{1}{2}$d(F_{e_2}, V(F_{e_1}, F_{e_2}, $\frac{1}{2}$))

From (i), d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = d(F_{e_2}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = $\frac{1}{2}$ d(F_{e_1}, F_{e_2})

This completes the proof.

Definition 3.6

Let (F_E, d, V) be a convex fuzzy soft metric space. A sequence $\{F_{e_n}\}$ in (F_E, d, V) is said to be converge to $F_e \in F_E$ if $\lim_{n\to\infty}$ d (F_{e_n}, F_e, λ) = 1, for all n >0.

Definition 3.7

Let (F_E, d, V) be a convex fuzzy soft metric space. A sequence $\{F_{e_n}\}$ is said to be Cauchy sequence if $\lim_{n\to\infty}$ d ($F_{e_{n+p}}$, F_{e_n}, λ) = 1, for all n >0 and p>0.

Definition 3.8

A convex fuzzy soft metric space (F_E, d, V) is said to be complete if every Cauchy sequence in F_E convergent to a fuzzy soft point in F_E.

Theorem 3.9 (Fixed Point Theorem on Convex Fuzzy Soft Metric Space)

Let F_A be a nonempty convex subset of a convex fuzzy soft complete metric space (F_E, d, V) and f be a self mapping of F_A. If there exists a, b, c, k such that

$$2b - |c| \le k < 2(a + b + c) - |c|,$$

$$a d(F_{e_1}, f(F_{e_1})) + b d(F_{e_2}, f(F_{e_2})) + c d(f(F_{e_1}), f(F_{e_2})) \le k d(F_{e_1}, F_{e_2})$$

for all F_{e_1}, $F_{e_2} \in F_A$, then f has at least one fixed point.

Proof

Let (F_E, d, V) be a convex fuzzy soft metric space.

Let F_A be a nonempty subset of a convex fuzzy soft complete metric space and F_{e_1}, $F_{e_2} \in F_A$

Assume that there exists a, b, c, k such that 2b - |c| ≤ k < 2(a + b+ c) - |c| and --- (1)

ad(F_{e_1}, $f(F_{e_1})$) + bd(F_{e_2}, $f(F_{e_2})$) + cd($f(F_{e_1})$, $f(F_{e_2})$) ≤ kd(F_{e_1}, F_{e_2}). --- (2)

To prove the self mapping f has at least one fixed point.

Suppose $F_{e_0} \in F_A$ is arbitrary.

A sequence $\{F_{e_n}\}_{n=1}^{\infty}$ is defined in such a way that

F_{e_n}= V($F_{e_{n+1}}$, $f(F_{e_{n-1}})$, $\frac{1}{2}$), n=1, 2. . .

As F_A is convex, $F_{e_n} \in F_A$ for all n∈ .

Since d(F_{e_1}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = d(F_{e_2}, V(F_{e_1}, F_{e_2},$\frac{1}{2}$)) = $\frac{1}{2}$ d(F_{e_1}, F_{e_2}), we have

d(F_{e_n}, $f(F_{e_n})$) = 2 d(F_{e_n}, $F_{e_{n+1}}$)

d($F_{e_{n-1}}$, $f(F_{e_{n-1}})$) = d(F_{e_n}, $F_{e_{n-1}}$) for all n ∈ .

Let c be a non negative number.

Then by triangle inequality,

2cd(F_{e_n}, $F_{e_{n+1}}$) - cd(F_{e_n}, $F_{e_{n-1}}$)≤ cd($f(F_{e_n})$, $f(F_{e_{n-1}})$)

Similarly for the case c < 0, we get

2cd(F_{e_n}, $F_{e_{n+1}}$) + cd(F_{e_n}, $F_{e_{n-1}}$)≤ cd($f(F_{e_n})$, $f(F_{e_{n-1}})$)

Therefore, for each case we have

2cd(F_{e_n},$F_{e_{n+1}}$) - |c|d(F_{e_n}, $F_{e_{n-1}}$)≤ cd($f(F_{e_n})$, $f(F_{e_{n-1}})$).

Now, by substituting F_{e_1} with F_{e_n} and F_{e_2} with $F_{e_{n-1}}$ in (2), we get

ad(F_{e_n}, $f(F_{e_n})$) + bd($F_{e_{n-1}}$, $f(F_{e_{n-1}})$) + cd(f(F_{e_n}), $f(F_{e_{n-1}})$) ≤ kd(F_{e_n}, $F_{e_{n-1}}$)

Therefore 2ad(F_{e_n}, $F_{e_{n+1}}$) + 2bd(F_{e_n}, $F_{e_{n-1}}$) + cd($f(F_{e_n})$, $f(F_{e_{n-1}})$) ≤ kd(F_{e_n}, $F_{e_{n-1}}$)

Substituting the value of cd($f(F_{e_n})$, $f(F_{e_{n-1}})$), we get

2ad(F_{e_n}, $F_{e_{n+1}}$) + 2bd(F_{e_n}, $F_{e_{n-1}}$) +2cd(F_{e_n}, $F_{e_{n+1}}$) - |c|d(F_{e_n}, $F_{e_{n-1}}$) ≤ kd(F_{e_n}, $F_{e_{n-1}}$)

⇒ 2ad(F_{e_n}, $F_{e_{n+1}}$) +2cd(F_{e_n}, $F_{e_{n+1}}$) ≤ kd(F_{e_n}, $F_{e_{n-1}}$) +|c|d(F_{e_n}, $F_{e_{n-1}}$)- 2bd(F_{e_n}, $F_{e_{n-1}}$)

⇒ 2[a +c] d(F_{e_n}, $F_{e_{n+1}}$) ≤ [k + |c| - 2b] d(F_{e_n}, $F_{e_{n-1}}$)

$$\Rightarrow d(F_{e_n}, F_{e_{n+1}}) \le \frac{[k + |c| - 2b]}{2[a+c]} d(F_{e_n}, F_{e_{n-1}})$$

From (1), $\frac{[k + |c| - 2b]}{2[a+c]} \in [0, 1)$ and hence $\{F_{e_n}\}_{n=1}^{\infty}$is a contraction sequence in F_A.

Therefore it is a Cauchy sequence.

Since $F_A$is a closed subset of a complete space, there exists $F_v \in F_A$ such that $\lim_{n\to\infty} F_{e_n} = F_v$, Therefore $\lim_{n\to\infty} f(F_{e_n}) = F_v$.

Now by substituting F_{e_1} with F_v and F_{e_2} with F_{e_n} in (2), we get

ad(F_v, $f(F_v)$) + bd(F_{e_n}, $f(F_{e_n})$) + cd($f(F_v)$, $f(F_{e_n})$) ≤ kd(F_v, F_{e_n}), for all n∈ .

Letting n ⟶ ∞, it follows that (a +c) d(F_v, $f(F_v)$) ≤ 0

Since a +c is positive, we have d(F_v, $f(F_v)$) = 0.

Therefore $f(F_v) = F_v$.

Hence the self mapping f has at least one fixed point.

This completes the proof.

Corollary 3.10

Let (F_E, d, V) be a convex fuzzy soft complete metric space and F_A be a nonempty subset of F_E. Suppose that f is a self mapping. If there exists a, b, k such that 2b ≤ k < 2(a+ b) and ad(F_{e_1}, $f(F_{e_1})$) + bd(F_{e_2}, $f(F_{e_2})$) ≤ kd(F_{e_1}, F_{e_2})for all F_{e_1}, F_{e_2}∈ F_A, then the set of all fixed point of f is a nonempty set.

Proof

Let (F_E, d, V) be a convex fuzzy soft complete metric space and F_A be a nonempty subset of F_E.

Assume that there exists a, b, k such that 2b ≤ k < 2(a+ b) and

ad(F_{e_1}, $f(F_{e_1})$) + bd(F_{e_2}, $f(F_{e_2})$) ≤ kd(F_{e_1}, F_{e_2}) for all F_{e_1}, F_{e_2}∈ F_A.

To prove that the set of all fixed point of f is a nonempty set.

Substitute c = 0 in the fixed point theorem of convex fuzzy soft metric space, we have

The self mapping f has at least one fixed point.

Hence the set of all fixed points of f is non empty.

References

Asadi, M. "Some Results of Fixed Point Theorems in Convex Metric Spaces." *Non Linear Functional Analysis and Applications*, vol. 19, no. 2, 2014, pp. 171-175.

Das, S. and Samanta, S.K. "Soft Metric." *Annals of Fuzzy mathematics and Informatics*, vol. 6, no. 1, 2013, pp. 77-94.

Majeed, S.N. "Some Notions on Convex Soft Sets." *Annals of Fuzzy Mathematics and Informatics*, vol. 12, no. 4, 2016, pp. 517-526.

Molodstov, D. "Soft Set Theory - First Results." *Computers & Mathematics with Applications*, vol. 37, no. 4-5, 1999, pp. 19-31.

Riaz, M. and Zain Fatima. "Certain Properties of Soft Metric Space." *The Journal of Fuzzy Mathematics*, vol. 25, no. 3, 2017, pp. 543-560.

Roy, S. and Samanta, T.K. "A note on Fuzzy Soft Topological Spaces." *Annals of Fuzzy Mathematics and Informatics*, vol. 3, no. 2, 2012, pp. 305-311.

Yildrim, E.D. et al. "On Soft I-baire spaces." *Annals of Fuzzy Mathematics and Informatics*, vol. 10, no. 1, 2015, pp. 109-121.

Zadeh, L.A. "Fuzzy Sets." *Information and Control*, vol. 8, no. 3, 1965, pp. 338-353.

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