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Problem-Solving Ability as a Predictor of Academic Achievement in Mathematics among Higher Secondary Students: A Structural Equation Modelling Approach

R. Manikandan
Bharathidasan University, India

A. Edward William Benjamin
Bharathidasan University, India

Abstract
Background: *Problem-solving skills have long been viewed as the core of learning mathematics, but empirical modelling of the multidimensional nature of problem-solving and its predictive role in mathematics achievement has been little evident in higher secondary education.*
Purpose: *This study hypothesises problem-solving skill as a second-order latent construct of numerical, logical, figural, and analytical dimensions and the predictive effect of the concept on the performance of mathematics in higher secondary students.*
Methods: *A cross-sectional design was used in this quantitative study. A standardised Problem-Solving Ability Test and a test on mathematics achievement were used to collect data on 300 higher secondary students in Tamil Nadu. The analysis of data was performed with the help of correlation analysis, regression analysis, Confirmatory Factor Analysis (CFA) and Structural Equation Modelling (SEM).*
Results: *A significant positive correlation was found between problem-solving ability and performance in mathematics (r = +.742, p <.001). The SEM model showed an excellent fit (kh2/df =2.28, CFI = .95, TLI =.94, RMSEA =.047). Adaptability to problems was an excellent predictor of mathematics success (b =.74, p <.001) and contributed 55% of the variance (*R^2*=.55).*
Conclusion: *The results support the hypothesis that problem-solving skills are a hierarchical cognitive structure and a good predictor of mathematics performance among high-level school learners.*
Future Scope: *Future studies should consider the structural model by adding motivational factors such as self-efficacy and mathematics anxiety, and longitudinal designs to investigate causal associations.*

Keywords: Problem-Solving Ability, Mathematics Achievement, Sem, Cognitive Skills, Latent Variables, Higher Secondary Students

Introduction

Mathematics education is important for enhancing learners' logical reasoning, analytical thinking, and problem-solving abilities. Problem-solving ability is one of the cognitive skills that have been cited in relation to mathematical learning and serves as a key element of mathematical proficiency. Classical views on the education of mathematics hold that problem solving is the intellectual heart of mathematics because it allows learners to use conceptual knowledge to solve problems in the unknown and create systematic problem-solving strategies (Schoenfeld, 1985). Modern education reforms still focus on problem-solving as a learning outcome in mathematics. Curriculum models are also becoming more focused on higher-order cognitive skills, such as reasoning, understanding concepts, and the strategic use of thinking, as key to effective learning of mathematics. Such views indicate that effective learning in mathematics is possible when students are involved in learning challenging tasks which require interpretation, reasoning, and assessment, as opposed to recalling procedural knowledge. Recent empirical studies continue to confirm that mathematical problem-solving ability is a key predictor of academic achievement and is closely linked to cognitive and motivational factors (Wang et al., 2025).

Mathematically, performance is strongly correlated with complex cognitive processes that can be defined as schema building, working memory, and metacognitive regulation (Mayer, 2002). These processes enable learners to solve mathematical scenarios, choose correct solution strategies, and keep track of how effective their reasoning is. As a result, learners who possess greater cognitive ability to solve problems are likely to show better mathematical performance. Although the issue of problem-solving in mathematics education has been identified as significant, numerous empirical studies have not articulated the issue of problem-solving ability as a multidimensional construct of cognition but as a singular observable variable. Mathematical problem-solving entails the combination of multiple cognitive dimensions, with numerical reasoning, logical deduction, spatial interpretation, and analytical thinking being the main dimensions involved. Further information about these dimensions and how they interact to produce academic performance may also add to our understanding of the mechanisms underlying mathematics performance.

Moreover, new opportunities for more rigorous research on these relationships have been provided by recent advancements in quantitative research methods. In particular, Structural Equation Modelling (SEM) enables researchers to test latent constructs and complicated relationships between a group of variables simultaneously (Kline, 2016). Nonetheless, there are only relatively few studies that utilise SEM to test the multidimensional nature of problem-solving skills and their capacity to affect mathematics performance, particularly in the setting of higher secondary schools in India.

Thus, the proposed research conceptualises problem-solving capability as a multidimensional latent variable and tests its predictive effects on mathematics performance among higher secondary learners through Structural Equation Modelling.

Review of Literature

The effect of mathematics self-belief and problem-solving skills toward creativity and achievement: Mathematics self-efficacy positively affects problem-solving skills which mediate the mathematical creativity and likely the mathematical performance in terms of achievement. This underscores the importance of confidence and problem-solving skills to math achievement (Asare et al., 2025).

The role of cognitive abilities and self-monitoring on academic achievement: Cognitive ability (logical reasoning and information processing) has significant influence on the students' academic achievement, including in Mathematics, and self-monitoring has a moderating effect on this as shown by the interaction between cognitive and metacognitive skills (Shi & Qu, 2022).

Effect of problem-solving skills and AI-supported teaching on mathematics achievement:AI-supported methods of teaching mathematics improve students' engagement in learning and mathematical problem-solving skills, which are essential to higher secondary school mathematics achievement (Ohene Boateng et al., 2026). Academic self-concept and academic achievement:Academic self-concept has a positive effect on academic achievement, suggesting that problem-solving skill might be affected by the motivational self-beliefs (Chen et al., 2014).

Math skills as a predictor for related STEM subjects: Problem solving performance in physics was found to be directly affected by different math skills, particularly algebraic skills, which indicates the fundamental nature of math problem solving skills (Tong et al., 2024). The relationships between motivation, self-efficacy, stress, and math achievement are moderated by gender and ability grouping:Ability grouping and gender differences account for the relationships between motivation, self-efficacy, stress, and math achievement, suggesting problem-solving ability might not work in the same manner across sub-groups (Kim et al., 2026).

The academic achievement is mediated by cognitive flexibility and critical thinking disposition, and math anxiety's role is somewhat variable; problem-solving skill is likely to be a factor that links these cognitive-affective factors (Gökçe & Güner, 2024).

The indirect effect of classroom management on mathematics achievement through mathematics student motivation is related to students' involvement in the problem solving tasks (Van Dijk et al., 2019) and that effective classroom management impacts students' motivation, which in turn influences students' mathematics achievement. Motivation factors related to math under-achievement and implications for STEM interest: Math motivation is a significant predictor of achievement gaps;Students' problem-solving skills may play a role in bridging the gap of underachievement and enhancing academic trajectories in the future for STEM (Fong & Kremer, 2019).

The negative impact of the dependency on AI tools on preservice teachers' problem-solving ability: Overreliance on artificial intelligence tools can hinder mathematics educators' problem-solving capabilities, which is essential for academic success, necessitating the development of independent problem-solving skills (Zhang et al., 2025).

Theoretical Framework

The conceptual framework of the current research is based on cognitive information processing theory and the constructivist learning paradigm, which are jointly used to explain the multidimensional nature of problem-solving ability and its impact on mathematics performance. In the cognitive information processing model, learning can be viewed as an active mental process that entails encoding, organising, integrating, and retrieving information (Mayer, 2002; Paas et al., 2003). Consequently, mathematical performance is determined by the effectiveness of learners in working with symbolic representations, creating schemas, and controlling working memory resources.

Based on this, problem-solving skills are the coordinated behaviours of multiple related mental elements. Numerical skills reflect quantitative processing and the activation of computational schemas. Logical ability is associated with deductive and inductive reasoning required to draw valid conclusions. Figural ability is visual-spatial processing, which is important for reading diagrams, graphs, and geometric relationships. Higher-order executive control processes, such as broken-down processes, assessment, and systematic reasoning, are indicated by analytical ability. These four dimensions are theoretically grounded in the cognitive frameworks of executive functioning and schema building (Diamond, 2013; Bull & Lee, 2014). Each dimension embodies a unique but interconnected mental activity that is necessary to solve a mathematical problem successfully.

Cognitive load theory explains the structural integration of these parts. According to Sweller, problem solving involves handling intrinsic cognitive load through schema automation. Students with greater numerical and logical processing capabilities are in a position to minimise extraneous processing requirements, and analytical skills increase strategic monitoring. Figural processing aids in dual coding and spatial representation, which enhances conceptual integration. These mental subsystems function as coordinated processes rather than individual capabilities. This theoretical combination explains why problem-solving ability is considered a higher-level latent construct rather than an independent variable. Constructivist learning theory is suitable for augmenting this explanation by underlining the fact that mathematical knowledge is built through interaction with the context of meaningful problems. To find a solution to non-routine problems, students must combine the processes of numerical computation, logical reasoning, spatial interpretation, and analytical reflection simultaneously. Hence, constructivist views lend credence to the conceptualisation of problem-solving ability as a composite cognitive system that arises from the interaction of its constituent dimensions. This synthesis of theory is illustrated in the multidimensional design shown in the conceptual diagram.

Constructivist learning theory complements this explanation by emphasising that mathematical knowledge is constructed through engagement with meaningful problem contexts. When students encounter non-routine problems, they must simultaneously integrate numerical computation, logical reasoning, spatial interpretation, and analytical reflection. Therefore, constructivist perspectives support the conceptualisation of problem-solving ability as an integrated cognitive system emerging from the interaction among its component dimensions. The multidimensional structure illustrated in the conceptual diagram reflects this synthesis.

Metacognitive theory further strengthens this hierarchical model. Schoenfeld (1985) demonstrated that effective problem-solving requires ongoing monitoring, planning, and evaluation. Analytical ability in the present model represents this regulatory function, serving as an integrative mechanism that coordinates numerical, logical, and figural processes. Empirical studies have confirmed that metacognitive regulation significantly predicts mathematics achievement (Kramarski & Mevarech, 2003). This supports the conceptualisation of problem-solving ability as a second-order construct that captures coordinated cognitive and metacognitive operations.

Socio-cognitive theory offers another level of explanation. According to the self-efficacy system developed by Bandura (1997), students' beliefs in their ability to reason determine their levels of engagement and persistence in complex tasks. Although self-efficacy does not have a direct model linking it to the current structural diagram, it acts as a contextual element that strengthens the activation of the four cognitive dimensions. Therefore, problem-solving capacity operates within a larger motivational-cognitive framework that defines achievement in mathematics.

Notably, current structural modelling studies affirm that cognitive skills are treated as latent variables that directly affect academic performance. The structural equation modelling approach enables the expression of problem-solving ability as a second-order latent variable measured with the help of numerical, logical, figural, and analytical first-order variables. This is a hierarchical structure that matches the conceptual diagram in Figure 1. In the structural part of the model, mathematics achievement is predicted by the higher-order construct; the theoretical consideration is that coordinated cognitive functioning directly causes the outcomes of academic performance. Cognitive information processing theory is used to describe the process by which each dimension works, constructivist theory to describe how these processes are integrated into meaningful tasks, metacognitive theory to describe how such coordination is regulated, and structural modelling methodology to provide an analytical description of how the integration is represented as a second-level latent construct. These points of view combined support the conceptual model in which problem-solving ability, which consists of numerical, logical, figural, and analytical dimensions, has a direct structural effect on mathematics achievement.

Conceptual Model

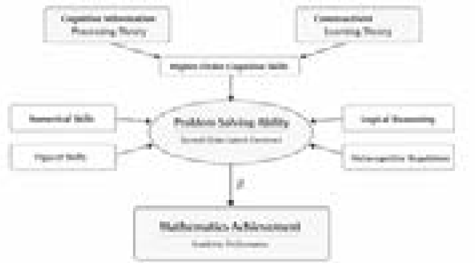

Figure 1

Conceptual framework illustrating problem-solving ability as a second-order latent construct comprising numerical, logical, figural, and analytical dimensions, grounded in cognitive information processing and constructivist learning theories, and its structural effect (β) on mathematics achievement.

Method

Research Design

The current research used a quantitative cross-sectional design with a normative survey design. The design was deemed suitable because the main aim of the study was to investigate the structural relationship between problem-solving ability and mathematics achievement without controlling for variables. The researchers wanted to model the latent constructs and predictive pathways with structural equation modelling (SEM), which necessitates data that were taken within natural educational settings.

Participants and Sampling Procedure

The sample comprised 300 higher secondary students in grades XI and XII from schools in Tamil Nadu, India. A stratified random sampling method was implemented to ensure sufficient representation of various demographic variables, such as gender, school type (government and private), and locality (urban and rural). Schools were initially classified into strata according to administrative classification. Institutional enrolment lists were used to randomly select participants from each stratum. The study sample of 300 was considered sufficient for the SEM analysis. Kline (2016) provides a minimum ratio of 10 participants per estimated parameter as the recommended minimum ratio for stable structural modelling. Because the second-order factor model was used in the current research, the sample size met the recommended statistical power. The participants' ages ranged from 16 to 18 years (M = 16.9, SD = 0.64). Data collection was performed voluntarily, and informed consent was obtained from the school authorities and students before data collection.

Instruments

Problem-Solving Ability Test

The ability to solve problems was assessed using a standardised problem-solving ability test that was designed and tested at the higher secondary level. The tool had 80 items that were allocated equally across four dimensions: Numerical Ability (20 items), Logical Ability (20 items), Figural Ability (20 items), and Analytical Ability (20 items). Items were constructed to assess quantitative reasoning, deductive and inductive reasoning, visual–spatial interpretation, and higher order analytical processing. Responses were scored dichotomously (1 = correct, 0 = incorrect), with higher scores indicating a stronger problem-solving ability.

The items were designed to evaluate quantitative reasoning, deductive and inductive reasoning, visual-spatial interpretation, and higher-order analytical processing abilities. The scoring of responses was based on a dichotomy (1 = correct, 0 = incorrect), and the higher the score, the better they were at solving problems. Content validity was determined by expert evaluation by five mathematics education and educational psychology experts. Confirmatory Factor Analysis (CFA) was used to evaluate construct validity, as it supported the four-factor first-order structure. Cronbach's alpha Coefficients of Internal Consistency reliability were as follows: Numerical Ability: a = .78; Logical Ability: a = .84; Figural Ability: a = .80; Analytical Ability: a = .86; Overall Scale: a =.91. These values are above the acceptable level of 0.70 suggested for educational research.

Mathematics Achievement Test

Mathematics achievement was measured using a standardised Mathematics Achievement Test, which was in accordance with the higher secondary curriculum. The test included procedural and application-based problems in algebra, calculus, trigonometry, and coordinate geometry. The tool also had good internal consistency (α =.88). Pilot testing was conducted to test item difficulty and discrimination indices to guarantee proper psychometric quality.

Data Collection Procedure

The collected data were gathered during normal school hours under standardised testing conditions. The participants were given instructions on how to perform the tests and were given a reasonable amount of time to complete the two instruments. All responses were anonymised to ensure confidentiality.

Data Screening and Preliminary Analysis

The dataset was filtered in terms of missing data, normality, and outliers before inferential analysis. Less than 2 per cent of the data were missing and were treated with expectation-maximisation estimation. The skewness and kurtosis values were within +-2, which implies that univariate normality is acceptable. Multivariate normality was evaluated using the Mardia coefficient. There were no instances of severe multicollinearity because the values of the variance inflation factor were less than 3.0. This type of diagnostic justified the appropriateness of the data in the SEM analysis.

Statistical Analysis

The data were analysed with the help of SPSS (Version XX) descriptive analysis, correlation analysis, and regression analysis, and AMOS (Version XX) Confirmatory Factor Analysis and Structural Equation Modelling. The product-moment Pearson correlation coefficient was calculated to test the nature and strength of the relationship between problem-solving ability and mathematics performance. Simple linear regression analysis was performed to estimate the proportion of variance explained. The assessment of problem-solving ability was confirmed through Confirmatory Factor Analysis (CFA). A second-order factor model was used, incorporating numerical, logical, figural, and analytical skills as components of a higher-order latent construct. The proposed structural relationship between problem-solving ability and mathematics achievement was subsequently analysed using Structural Equation Modelling (SEM). The estimation process employed maximum likelihood, and a chi-square to degrees of freedom ratio (χ^2/df) of ≤ 3.0 was deemed acceptable.

Comparative Fit Index (CFI) and Tucker-Lewis Index (TLI) values $\geq .90$ indicated good fit.

Root Mean Square Error of Approximation (RMSEA) $\leq .08$ and Standardised Root Mean Square Residual (SRMR) $\leq .08$ were considered acceptable.

Ethical Considerations

Data collection was performed with the ethical approval of the institutional research committee. The research was conducted on a voluntary basis, and the responses were kept confidential. Participation did not influence the students' academic assessment.

Results

Preliminary Analyses

Data were screened before hypothesis testing. The skewness and kurtosis values indicated no substantial deviations from normality ($|\text{skewness}| < 1.0$; $|\text{kurtosis}| < 1.0$). No multivariate outliers were detected based on the Mahalanobis distance ($p > .001$). The descriptive statistics and bivariate correlations are presented in Table 1. Problem-solving ability demonstrated a strong positive association with mathematics achievement ($r = .742$, $p < .001$), indicating a large effect size.

Table 1 Descriptive Statistics and Correlation Matrix

Variable	M	SD	1	2
Problem-Solving Ability	112.48	14.62	-	.742***
Mathematics Achievement	78.36	10.94	.742***	-

Note: N = 300, $p < .001$.

A preliminary linear regression analysis was conducted to examine the predictive strength of problem-solving ability on mathematical achievement. The model was statistically significant, $F(1, 298) = 364.87$, $p < .001$, explaining 55% of the variance in mathematics achievement, $R^2 = .55$ (Adjusted $R^2 = .548$). Problem-solving ability significantly predicted mathematics achievement ($\beta = .742$, $t = 19.10$, $p < .001$; see Table 2).

Table 2 Regression Analysis Predicting Mathematics Achievement

Predictor	B	SE B	β	T	p
Problem-Solving Ability	0.56	0.03	.742	19.10	< .001

Note: R=.742, R^2=.550, Adjusted R^2=.548 F(1,298)=364.87, p < .001

Measurement Model

A second-order Confirmatory Factor Analysis (CFA) was conducted using maximum likelihood estimation to examine the hypothesised hierarchical structure of problem-solving ability, comprising four first-order latent factors: numerical, logical, figural, and analytical abilities.

The Measurement Model Demonstrated Good Fit to the Data

$\chi^2(224) = 517.44$, $\chi^2/df = 2.31$, CFI = .95, TLI = .94, RMSEA = .048, 90% CI [.041, .056], and SRMR = .041.

All standardised factor loadings were statistically significant ($p < .001$) and ranged from .69 to .87, supporting the convergent validity. The composite reliability (CR) values ranged from .81 to .88, exceeding the recommended threshold of .70. The average variance extracted (AVE) values ranged from .52 to .61, indicating adequate construct reliability.

Table 3 Confirmatory Factor Analysis: Standardized Factor Loadings

Factor	Indicator Range	Standardized Loading	CR	AVE
Numerical Ability	NA1–NA20	.69–.82	.83	.55
Logical Ability	LA1–LA20	.72–.85	.86	.58
Figural Ability	FA1–FA20	.69–.78	.81	.52
Analytical Ability	AA1–AA20	.74–.87	.88	.61

Note: All loadings significant at $p < .001$.

Structural Model

Structural equation modelling (SEM) was conducted to test the hypothesised direct effect of problem-solving ability on mathematics achievement. The structural model demonstrated a good fit to the data.

$\chi^2(226) = 515.84$, $\chi^2/df = 2.28$, CFI = .95, TLI = .94, RMSEA = .047, 90% CI [.040, .055], and SRMR = .043.

Problem-solving ability significantly predicted mathematics achievement ($\beta = .74$, SE = .05, CR = 14.80, $p < .001$; see Table 4). The model accounted for 55% of the variance in mathematics achievement ($R^2 = .55$).

Table 4 Structural Equation Modeling Results

Path	β	SE	CR	P
Problem-Solving Ability → Mathematics Achievement	.74	.05	14.80	< .001

Discussion

The current research study explored problem solving ability as a second-order latent construct, which was multidimensional and examined its structural effects on mathematics achievement among the high secondary students. The results have robust empirical evidence to the main hypothesis that problem-solving ability is a dominant explanatory construct to predict mathematics achievement. The results have robust empirical evidence to the main hypothesis that problem-solving ability is a dominant explanatory construct to predict mathematics achievement. The current findings align with recent studies showing that problem-solving skills are a crucial factor in forecasting mathematics performance in various educational settings (Hiltrimartin & Pratiwi, 2025). Additionally, modern frameworks highlight that the cognitive and executive processes involved in problem-solving can be accurately modeled using structural equation modeling techniques. The large correlation (r = .742, p-value = .001)... The large correlation (r =.742, p-value =.001) and structural path coefficient (b =.74, p-value =.001) are indicative that higher-order cognitive competencies are the intellectual basis of mathematical performance and not the peripheral academic capabilities. This observation is in line with traditional notions of mathematical problem solving, where reasoning, strategy use, and reflective thinking are the primary determinants of success (Schoenfeld, 1985).

The size of the explained variance (R^2 =.55) indicates that over half of the variability in mathematics achievement can be explained by variation in the problem solving capability of the students. This is a significant amount of explanatory power in the field of educational research, where achievement results are generally affected by numerous cognitive, motivational and situational variables. It strengthens the opinion that mathematical performance is closely embedded in organized cognitive processing systems such as reasoning efficiency, schema building as well as metacognitive regulation (Sweller, 1988). This result is also consistent with global standards, where problem solving and reasoning are placed at the core of mathematical proficiency.

In terms of measurement, confirmatory factor analysis validated a second-order latent structure, which made the problem-solving ability construct valid. The first four-order dimensions, including numerical, logical, figural, and analytical ability, exhibited high factor loadings and mediocre reliability coefficients, which justified the fact that problem-solving ability could be conceptualised as a hierarchically organised cognitive system, but not a single trait. The comparatively higher loadings of the dimensions of analytical and logical reasoning indicate that abstract reasoning and organised thinking make the greatest contribution to the mathematical performance at the higher secondary level. This is theoretically in line with developmental and cognitive views, where higher levels of mathematical involvement are aligned with formal operational reasoning and symbolic abstraction (Schoenfeld, 1985). The results also support the cognitive information processing theory, which assumes that the achievement of learning is based on the effectiveness of the encoding, organisation, and retrieval of information by learners (Sweller, 1988). Students who demonstrate higher analytical and logical reasoning skills are more capable of sorting mathematical information into logical schemas, have a better understanding of intrinsic cognitive load, and can implement an appropriate solution strategy. The high level of structural correlation observed using SEM empirically supports the theoretical assertion that systematic cognitive processing is a direct mediator of success in areas typified by abstraction and symbolic representation.

Constructivist views of learning mathematics are also supported. Constructivist theory focuses on the fact that knowledge building occurs as a result of active involvement in problem situations that have some meaning. The high predictive value of the current study implies that students who are successfully involved in systematic problem-solving activities show high academic achievement. This observation favours instructional methods where logic-based learning, engagement with non-routine problems, and conceptual logic are all taught as the main instructional techniques (NCTM, 2000).

The current coefficient (b = .74) is slightly higher than the predictive power of SEM-based studies with structural coefficients ranging from b =.65 to b =.72. This implies the contextual strength of problem-solving-achievement unity in the higher secondary system under investigation. The relatively high effect size can be attributed to the highly analytic nature and examination-focused academic culture of the high secondary level, where reasoned thinking and organised problem-solving are prioritised.

The study methodologically builds on previous work on the topic by utilising second-order structural equation modelling instead of just using bivariate correlations or regression analysis. SEM provides the ability to estimate measurement and structural components simultaneously and consider measurement errors, thereby improving precision and construct validity. Such methodological rigor enhances the validity and generalisability of the results and is in line with current quantitative requirements in educational research.

Theoretically, a combination of cognitive, constructivist, and metacognitive viewpoints in the SEM framework provides a consistent explanation of mathematics performance. Problem-solving skills are a higher-order cognitive concept that involves a combination of reasoning functions, executive control, schema formation, and strategic control (Bandura, 1997; Schoenfeld, 1985). The conceptual framework of the study was confirmed by the significant structural path and indicated the correspondence of theoretical assumptions and empirical modelling.

The implications of the findings are of educational value. When the proportion of variance explained by problem-solving ability exceeds half of the overall variance in mathematics performance, instructional activities need to emphasise methodical building of reasoning, analytical cognition, and tactical involvement in problems. Assessment mechanisms also need to go beyond procedural recall by measuring multidimensional problem-solving capabilities (NCTM, 2000). Instruction in explicit strategies and metacognitive scaffolding should be introduced into teacher education programs to develop higher-order cognitive skills.

At the policy level, the results can be used to support competency-based education models that predict reasoning and critical thinking as key learning outcomes. Higher secondary curriculum reforms need to focus on structuring problem-solving interventions and reasoning-based instructional design, as opposed to rote memorisation. The study has limitations, even though it has its strengths. The cross-sectional design limits strong causal inference, but SEM has strong structural evidence. Longitudinal studies may also be required to determine whether the predictive relationship remains stable over time. In the future, studies can incorporate constructs of motivation, such as self-efficacy which has been reported to affect academic achievement (Bandura, 1997), or mathematics anxiety, to determine whether there are mediating or moderating roles under an enlarged structural framework.

Finally, the current experiment empirically confirms that problem-solving ability is a second-order latent construct that has a significant structural effect on mathematics outcomes. The findings combine the classical view of theoretical approaches and the modern SEM approach to provide strong evidence that the intellectual essence of mathematics is based on organised reasoning and strategic problem-solving and not on the acquisition of procedures alone.

Suggestions

Based on the research results, some practical and educational conclusions can be suggested. First, mathematics teaching at the higher secondary level must focus on problem-solving activities based on structure and not memorisation of procedures. Classroom activities created by teachers must also promote critical thinking, logical investigation of mathematical problems, and reasoning. Second, multidimensional problem-solving aspects, such as numerical reasoning, logical reasoning, figural interpretation, and analytical reflection, should be incorporated by curriculum developers into the mathematics curriculum. Such integration would assist students in acquiring the complete cognitive ability required for mathematical proficiency. Third, the pedagogy of problem-solving and metacognitive instruction methods must be trained in teacher education programs. When provided with such strategies, teachers can help students plan, track, and assess the processes of solving problems more effectively. Fourth, assessment systems need to be out of the traditional procedural type of tests and should also include problem-solving-based evaluation methods that are able to measure the higher-order cognitive aspects. Lastly, policymakers should be encouraged to endorse competency-based education reforms that focus more on reasoning, critical thinking, and conceptual understanding as learning outcomes in mathematics education.

Future Research Directions

Future research could build upon the current structural model by including motivational and emotional factors, such as self-efficacy, math anxiety, and learning strategies, to explore their mediating or moderating effects. Longitudinal research designs might also be utilised to identify causal relationships and developmental trends in problem-solving skills and mathematical performance. Furthermore, comparative studies in various educational settings and geographical areas will improve the applicability of the results.

Conclusion

The current study provides empirical evidence that problem-solving ability is a higher-order cognitive variable with a significant structural effect on mathematics performance among higher secondary school students. The study has developed both theoretical clarity and methodological rigor in mathematics education research because it conceptualises problem-solving ability as a multidimensional second-order latent variable, including numerical, logical, figural, and analytical dimensions. The high structural path coefficient ($\beta = .74$, $p <.001$) and the significant value of the explained variance ($R^2 =.55$) support that problem-solving ability is not a fringe academic skill but a core determinant of mathematics performance. The results add to the theoretical synthesis of cognitive information processing, constructivist learning theory, and metacognitive regulation into a single structural framework.

The findings support the hypothesis that the constructive development of schemata, systematic thinking, and procedural intelligence control is a precondition for success in mathematics, especially at the higher secondary level, when abstract thinking and the manipulation of symbols are prioritised. The structural equation modelling of the confirmation of a second-order measurement model provides additional support for the construct validity of problem-solving ability and shows the suitability of hierarchical cognitive modelling for educational research. The second-order SEM employed in this study to concurrently estimate measurement and structural elements, as well as consider measurement error, also answers the methodological questions of the research: it provides greater rigor in the quantitative methods used in educational psychology. This will increase the accuracy, consistency, and applicability of the results and place the study in the realm of modern analytical standards that are sought after in high-impact journals (Q1 and Q2). Educationally, the findings reinforce the need to focus more on higher-order cognition development in mathematics programmes. Teaching methods should shift toward systematic problem solving, analytical thought, and explicit metacognitive strategy teaching. Assessment systems also need to be based on multidimensional problem-solving competencies, as opposed to single procedural accuracy.

The results provide empirical support for competency-based remedies, where foreground reasoning and critical thinking are the learning outcomes. Although cross-sectional research does not allow causal inferences, the strength and consistency of the structural associations provide powerful evidence of the central position of problem-solving ability in academic success. This line of research can be pursued in future studies with a longitudinal or experimental research design to investigate developmental patterns and identify possible mediating variables such as motivation, self-efficacy, and mathematics anxiety. It would be useful to add affective and contextual variables to the structural model to obtain a more comprehensive picture of mathematics learning processes.

Overall, it is accurate in this study that the intellectual essence of mathematics achievement is organised reasoning and deliberate engagement with strategic problems. This study makes a significant contribution to the theory of mathematics education, measurement, and practice by empirically validating problem-solving ability as a multidimensional higher-order construct with a high degree of explanatory power. The results can be used to base further investigation and provide clear guidelines in terms of instructional innovation and policy changes that can be used to support higher-order cognitive skills in the teaching of mathematics.

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Author Details

R. Manikandan, *Bharathidasan University, India*

Dr. A. Edward William Benjamin, *Bharathidasan University, India*