Thirugnana Sambandhar - A Mathematician

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Abstract

India has been the Land of notable poets whose exemplary works are world renowned. One such great poet is Thirugnana Sambandhar. He is a saint, poet, philosopher, composer who belongs to 7th Century. He was born in Seerkaazhi of Tamilnadu. He had coined many Special Geometrical poetic structures like Thiru ezhukkootrirukkai (poem with mathematical Triangular Pattern), Maalai Maatru (a poem with palindromic Structure), Mozhi Maatru (a poem in which the meaning of the poem can be observed by a systematic Chane of words), Gomuthri (Flow of the poem in such a way it forms a wave line), Chakramaatru (a poem which is constructed in a circular form). By the above mentioned amazing structure He has no parallels in the worlds poetry Thirugnana Sambandhar is the epitome of Tamil Literature has penned down many such extraordinary poems. A Mathematician is one who uses an extensive knowledge of Mathematics in their work. Mathematicians are concerned with numbers, data, quantity, structure, space,models and change. Here in this poetic form Thiruezhukkootrirukkai Thirugnana Sambandhar had used numbers in a brilliant way to form a Triangle. This is called "Chitrakavi" in Tamil. By analyzing the whole poem we will get a geometrical structure. In this Thiruezhukkootrirukkai Thirugnana Sambandhar has constructed the words in such a way to form a symmetrical triangle. These triangle is arranged in a perfect mathematical calculation. This can be analysed through the law of binomial co- efficient. This is analysed and proved in this paper. Thirugnana Sambandhar belongs to 7th Century whereas the Scientist and Mathematician Pascal who discovered the law of Bi-nomial co-efficient belongs to 17th century. Other than this Mathematical diagram of triangle this poem has Palindromic numbers which add more beauty to this structure which is also a mathematical calculation. By constructing this amazing poetic structure Thirugnana Sambandhar proves beyond doubt that he is a "Mathematician" of India of the 7th Century itself who had applied the law of triangle earlier.

Keywords: Chitra Kavi, Law of Binomial Co-efficient, Multifaceted Personality, Symmetrical Triangle, Thiruezhukkootrirukkai

Introduction

Thirugnana Sambandhar is a Multifaceted personality with profound knowledge about many subjects like Mathematics, Physics, Chemistry, Plant Sciences, Animal Sciences, Geology, Geography, Music, Dance, and so on. He has an insight knowledge in every subject and shows astounding knowledge on subjects like Mathematics, Science and so on. One such contribution is "Thiruezhukkootrirukkai". Here in this composition we can identify many numbers. These numbers shows some astonishing Arithmetic Formations which are identified in later period by many scientists.

Thiruezhukkootrirukkai

This a new poetic structure constructed by Thiruganana Sambandhar. This can be split into Thiru + Ezhu + Kootru + Irukkai. Thiru is the General Term for excellence. Ezhu = Number seven, Kootru = saying, Irukkai = formation. In Tamil literature this is a special type of verse. It is one among a variety of verses seen under "Chitrakavi". The word occurring in these verses can be represented aesthetically, appealing and familiar in a geometric form, a "Chariot" form.

The whole Thiruezhukkootrirukkai Padhigam has 47 Lines. In these Padhigam the numbers 1,2,3,4,5,6,7 are depicted in an order to form a Chariot This can be otherwise called as a Triangular Form. Like the apex of a Triangle the centre top will be sharp and from there will be a deviation on both sides to form a Triangle or Prism like structure. Thirugnanasambandhar was the first person to compose this special verse. Later following him Thirumangai Aalwar, Saint Nakkeerar and Arunagirinaadhar are among those who have composed in the same way. The two dimensional Chariot representation of these verses display the following numerical triangle pattern.

```
        1
      1 2 1
    1 2 3 2 1
  1 2 3 4 3 2 1
1 2 3 4 5 4 3 2 1
```

Pascal the Mathematician

Gnana Sambandhar Thiruezhukkootrirukkai is compared to Pascal's Law of Triangle. The Scientist Elaise Pascal belongs to 17th Century (1623-1662). He was a French Philosopher and Scientist. He was also one of the greatest and most influential mathematician of all times. Elaise Pascal in his short 39 years of life made many contributions and inventions in several fields. In Mathematics he is known for contributing Pascal's Triangle and Probability theory.

Pascals Triangle

To build a Triangle Pascal started with "1" at the Top. Then continue placing Numbers below it in a Triangular Pattern. Pascal's Triangle is a Never Ending Equilateral Triangle of Numbers that follow a rule of adding the two numbers above to get a Number below. The two sides of the triangle are "all number 1s" and because the triangle is infinite there Is no bottom side. But in Thiruezhukkootrirukkai, since seven is the last number we have bottom line.

Pascal's Law of Triangle

11^0 - 1
11^1 - 1 1
11^2 - 1 2 1
11^3 - 1 3 3 1
11^4 - 1 4 6 4 1
11^5 - 1 6 1 0 5 1
11^6 - 1 7 7 1 5 6 1
11^7 - 1 9 4 8 7 1 7 1

1 >>>>>>>>>>>>>>> → Straight Natural Numbers
1 2 1 >>>>>>>>>>>> → Triangular Numbers
1 3 3 1 >>>>>>>>>>> → Tetra Hedral Numbers
1 4 6 4 1 >>>>>>> → Pentatope Numbers
1 5 10 10 5 1 >>>>> → Five Simplex Numbers
1 6 15 20 15 6 1 >>> → Six Simplex Numbers
1 7 21 35 35 21 7 1 >> → Seven Simplex Numbers

Equation for Bi-nomial Expansion by Pascal

Pascals Triangle determine the coefficients which arise in Bi-nomial Expansion.

For Example consider the expansion.

$(x+y^2) = x^2 + 2 + y + y^2 = 1 \, x^2y^0 + 2 \, x^1y^1 + 1 \, x^0y^2$

The Co-efficients are the numbers in row two of Pascals Triangle 1,2,1. Gnana Sambandhars Thiruezhukkootrirukkai also starts with 1,2,1. It is a structure where the numbers one to seven are gradually increased in an order and then decreased to 1 in the same order. This is a new structure contributed by Thirugnanasambandhar to the "Poetic and Music World".

Analysation of the Song Thiruezhukkootrirukkai

The analysation is purely based on keen observation. The visual representation of numbers (Tamil Words) are seen in this poetic lines in a particular order (Ascending or Descending) to form a Triangular structure. So it is other wise called as "CHITRA KAVI".

Thiruezhukkootrirukkai - Song

Pann; Viyazha kurinji (Ragam - Sowrashtram)
Ooruruvaayenai maanaangaarath - 1

(Descending 2-1)
Theeriyal baayoru vinmudal boodhalam – 2-1
(Ascending 1-2)
Onedriya Irusudar Umbarkal Piravum – 1-2

(Descending 3-2-1)
Padaithalithazhippamum Moorthigalaayinai Eruvaro doruva raagi nindranai – 3-2-1

(Ascending 1-2-3)

Oraa - neezhal onkazhal erandum muppozhuthetia – 1-2-3

(Descending 4-3-2-1)

Naalvark kolineri Kaatinai naatta moondraahak kotinai Erunadhi arava modoru madhi soodinai – 4-3-2-1

(Ascendig 1-2-3-4)

Oru thaaleerayin moovilai soolam Naarkaal maan mari – 1-2-3-4

(Descending 5-4-3-2-1)

Aindalai aravam Endhinai Kaaindha naalvaai mummadha Thirukot torukari Eedazhi, thurithanai – 5-4-3-2-1

(Ascending 1-2-3-4-5)

Oruthanu Irukaal valiya vaangi muppurathodu naanilam anja – 1-2-3-4-5

(Descending 6-5-4-3-2-1)

Kondru thalathura avunarai aruthana Aimpulan naalam andhak karanam mukkunam eruvali orungia vaanor – 6-5-4-3-2-1

(Ascending 1-2-3-4-5-6)

Etha nindranai Orungiya manathodu eru pirap porndu muppozhuthu kurai mudithu naanmarai yodhi Aivagai velvi amaiththaaranga mudhalezhuth thodi – 1-2-3-4-5-6

(Descending 7-6-5-4-3-2-1)

Varanmurai payindrezhu vaandranai valarkkum piramapuram peninai Arupadham muralum venupuram virumbinai Igaliya maindh unar pugali yamardhanai Pongu naarkadal soozh venguru vilanginai Paani Moovulagum pudaiya mel midhandha Thonipurath thuraindhanai tholiyaa virunidhi - Vaaindha poondaraai eidhanai Parapuram endrunar sira pura thuraindhanai – 7-6-5-4-3-2-1

(Ascending and Descending 1-2-3-4-5-6-7-6-5-4-3-2-1)

Oru malai edutha Iru thiral arakkan – 1-2

Viral koduth tharulinai puravam purindhanai Munneer thyindron naanmugan ariyaa – 3-4

Panpodu nindranai sambi yamardhanai Aiyurum amanarum aruvagai therarum – 5-6

Oozhiyum unaraa kaazhi yamardhanai Echan Ezisaiyon kochaiyai mechinai – 7

Aarupadamum, Aindhamar kalviyum – 6-5

marimudal naangum – 4

moondru kaalamum thondra nindranai – 3

Irumaiyin orumaiyum orumayin perumaiyum – 2-1

Maruvilla maraiyor

Kazhumala mudhupathi kavuniyan katturai

Kazhumala mudhupathi kavuniyan ariyum

Aniya thanmaiyai aathalin ninai

Ninaiya vallva rillai neelnilathe

The Number Representation

There are 47 lines in this Thiruezhukkootrirukkai Padhigam. In each line the numbers are underlined. They are in the form of just word which add meaning to the song. The numbers are represented in the form of Tamil words as below.

Numbers	Words represented
No 1.as	Oru, mudal, ondru, oruvar, oraa, orumai
No 2.as	Eer, Iru, Iruvar, Irandu
No 3.as	Mun, mup, moondra, moo, munnir, moondru
No 4.as	Naal, naarkaal, naanilam, naalam, naan, naangum
No 5.as	Aindu, anju, aim, aivagai, aindhu,
No 6.as	Aaru, aru
No 7.as	Ezhu

The Structure of Thiru Ezhukkootrirukkai

```
            1
          2 1  1 2
        3 2 1  1 2 3
      4 3 2 1  1 2 3 4
    5 4 3 2 1  1 2 3 4 5
  6 5 4 3 2 1  1 2 3 4 5 6
7 6 5 4 3 2 1  1 2 3 4 5 6 7
```

Analysation of Triangles of Sambandhar Equilateral Triangle

In Sambandhar's Thiruezhukkootrirukkai shows two dimentional Triangle representation as shown above that is we can see two Triangles merged together to form Equilateral Triangle in the form of a Chariot Doom. Here the number one (1) forms the Chariots Apex. From there in both directions the numbers increases from one to seven (1 to 7) on both sides. As there are seven steps and the number

seven (7) is the last number of the expansion which is called Thiruezhukkootrirukkai.

Symmetrical Triangle

The Triangle formed in Thiruezhukkootrirukkai is also called Symmetrical Triangle. The numbers on the left sid e have identical matching numbers on the right side like a mirror image and it is also with

Palindromic Numbers like 1221, 321123.

```
        1
      2 1 1 2
    3 2 1 1 2 3
  4 3 2 1 1 2 3 4
```

Some more Sambandhar's Triangle according to Pascals Law:

1	1	$1^1 = 1$	1x1 = 1
121	1 2 1	$2^2 = 4$	11x11 = 121
12321	1 2 3 2 1	$3^2 = 9$	111x111 = 12321
1234321	1 2 3 4 3 2 1	$4^2 = 16$	1111x1111 = 1234321
123454321	1 2 3 4 5 4 3 2 1	$5^2 = 25$	11111x11111 = 123454321

In Horizontal way

They double each time (Power of Two). The above Triangle 1, 121, 12321, 1234321 becoming double in each line ie $1^2 = 1$, $2^2 = 4$, like that

```
1                →  1^1 = 1
1 2 1            →  2^2 = 4
1 2 3 2 1        →  3^2 = 9
1 2 3 4 3 2 1    →  4^2 = 16
```

Exponents of 11
Each Line also Proves Exponents of 11

```
1x1 = 1          ----------- 11^1
11x11 = 121      ----------- 11^2
111x111 = 12321  ----------- 11^3
```

Properties of Pascal's Triangle with that of Gnana Sambandhar

1. Each Number is a sum of two numbers above it.

```
        1
    (1) …. (1)
      1 (2) 1
```

In the above Triangle number 2 in the third row is the sum of two numbers (1) + (1) in the second line above.

2. The outside numbers are all 1.

```
        1
      1 2 1
    1 2 3 2 1
  1 2 3 4 3 2 1
1 2 3 4 5 4 3 2 1
```

In the above Triangle the outside numbers are 1.

3. The Triangle is symmetric.

```
        1
      2 1 1 2
    3 2 1 1 2 3
  4 3 2 1 1 2 3 4
```

The Number on the left side have identical matching Numbers on the right side like a mirror image.

4. The first Diagonal shows the counting Numbers. Here below 1,2,3,4 forms the counting Numbers or Natural Numbers.

```
        1
      2 1 1 2
    3 2 1 1 2 3
  4 3 2 1 1 2 3 4
```

5. The sums of the rows give the power of two.

```
1                →  1^1 = 1
1 2 1            →  2^2 = 4
1 2 3 2 1        →  3^2 = 9
1 2 3 4 3 2 1    →  4^2 = 16
```

6. Each row gives the digits of the power of 11.

```
11 x 11   = 121 (11^2)
111 x111 = 12321 (11^3)
1111 x 1111 = 1234321 → (11^4)
```

Conclusion

Thus Pascal's Law of Triangle is explained in the form of Poetic Structure by Gnana Sambandhar who belongs to seventh Century (Prior to Ten Centuries of Pascal). The starting of this Thiruezhukkootrirukkai is number one (Oruru) followed by one-two-one (1 2 1) later as said by Pascal. We are getting a perfect Triangle through this Thiruezhukkootrirukkai which

is in a triangular form, a special Poetic Structure. So Gnana Sambandhar becomes the First Mathematician, Scientist, Forerunner to introduce the Law of Triangle through his Thiruezhukkootrirukkai which forms the Prime source for the law of Triangle. The above research paper falls under Applied Science - Mathematics.

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