Weak Convex Domination in Hypercubes

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Abstract

The n-cube Q_n is the graph whose vertex set is the set of all n-dimensional Boolean vectors, two vertices being joined if and only if they differ in exactly one coordinate. The n-star graph S_n is a simple graph whose vertex set is the set of all n! permutations of {1, 2, • • • , n} and two vertices α and β are adjacent if and only if α(1)≠β(1) and α(i) ≠β(i) for exactly one i ,i≠ 1.

In this paper we determine weak convex domination number for hypercubes. Also convex, weak convex, m - convex and l1-convex numbers of star and hypercube graphs are determined.

Keywords: Convexity number, Weak convexity number, Weak convex domination, m - convexity number, l1 - convexity number.

Mathematics Subject Classification: 05C12

Introduction

Graphs considered here are connected, simple. Akers and Krishnamurthy introduced the n-star graph S_n [1]. The vertex set of Snis the set of all n permutations of {1, 2, • • • , n} and two vertices α and β are adjacent if and only if α(1) ≠β(1) and α(i) ≠β(i) for exactly one i, i ≠ 1.

The n-star graph is an alternative to n-cube with superior characteristics. Day and Tripathi have compared the topological properties of the n-star and the n-cube in [5]. Arumugam and Kala have determined some domination parameters of star graph and obtained bounds for γ, γ_i, γ_t, γc and γ_p in n-cube for n ≥ 7 in [2].

Let G be a simple connected graph. A subset S of V is called a convex set if for any u, v in S, S contains all the vertices of every u − v geodesic in G. A subset S of V is called a weak convex set if for any u, v in S , S contains all the vertices of a u − v geodesic in G.

A subset S of V is called a m - convex set if for any u, v in S,S contains all the vertices of every u − v induced path in G.

A subset S of V is called al1 - convex set if it is convex and has a vertex which is adjacent to rest of the vertices of S. Maximum cardinality of a proper convex set is the convexity number of G. In a similar way we define weak convex number, m - convex number and l_1 - convex is the maximum of {Con < N[x] > /x εV(G)}.

A subset S of V is called a domination set if every vertex in V − S is adjacent to at least one vertex in S. A dominating set is a weak convex dominating set if it is weak convex. So far exact value of domination number for large n in Q_n has not been determined. Here we determine weak convex domination number of Q_n for any n.

Results on Convexity Number Parameters

For Q_n

$Con(Q_n) = 2^{n-1}$ for all n.

$wcon(Q_n) = 2^{n-1}$ for all n.

$l_1 - con(Q_n) = 2$ for all n.

$mcon(Q_n) = 2^{n-1}$ for all n.

For S_n

$Con(S_3) = 3$.

$wcon(S_3) = 4$.

$mcon(S_3) = 2$

$l_1 - con(S_3) = 2$.

$Con(S_4) = 6$.

$wcon(S_4) = 18$.

$mcon(S_4) = 6$

$l_1 - con(S_4) = 3$.

In general

$Con(S_n) = 6$ for all n .

$wcon(S_n) = 2n$ for all n.

$mcon(S_n) = 6$ for all n

$l_1 - con(S_n) = n$ for all n, since girth of S_n is C_6 and circumference of S_n is C_{2n}. S_n is n-1 regular.

Main Results

Weak convex domination number in Q_n

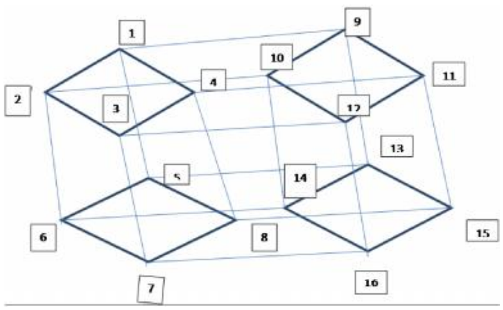
Theorem 2.1 $\gamma_{wc}(Q_n) = 2^{n-1}$for all n.

Proof: Q_n is a connected graph. For n = 1 ,$Q_1 = K_2$ and $\gamma_{wc}(Q_1) = 1 = 2^{1-1}$.

For n = 2,$Q_2 = C_4$ and $\gamma_{wc}(Q_2) = 2 = 2^{2-1}$.

For n = 3, $\gamma_{wc}(Q_3) = 4 = 2^{3-1}$.

For n = 4 the structure of Q_4 is given below.



We know that $\gamma(Q_3) = 2$. Either {3, 5} or {1, 8} can be chosen that is diametrically opposite vertices are chosen. Therefore their distance is three and hence $\gamma_{wc}(Q_3) = 4$. Let $\gamma_{wc}(Q_3)$ set be {1, 3, 4, 5}= A (say). A dominates {9, 11, 12, 13}. Therefore vertices to be dominated in Q4 are {10, 14, 15, 16}.Minimum two vertices are required to dominate {10, 14, 15, 16}. Therefore {1, 3, 4, 5, 14, 16} dominate Q_4, but they do not form a weak convex dominating set since geodesic between {1, 4} require 6 and geodesic between {3, 14} require 8. Thus minimum eight vertices are needed for a weak convex dominating set in Q_4. These eight vertices can be chosen in any manner from the two layers of Q_3. Hence we observe that for Q_n, 2^{n-1} vertices are required for a weak convex dominating set which can be got in any manner from the two layers of Q^{n-1}. Now we claim that 2^{n-1} is the minimum number of vertices for a weak convex dominating set in Q_n.

Let $k + l = 2^{n-1}$ where k,lare the number of vertices chosen in two layers of $Q_{n-1}$for a weak convex dominating set in Q_n.

Without loss of generality assume $l < k$. Let Q^1_{n-1} and Q^2_{n-1} denote the first and second layers of Q_{n-1}. Choose k vertices in Q^1_{n-1}in such a way that they form a weak convex dominating set in Q_{n-1}. Suppose we take $l - 1$ vertices in Q^2_{n-1}. Now we claim that $k + l - 1$ vertices do not form a weak convex dominating set in Q_n.

Case (i)

$l - 1$ vertices are private neighbors of k vertices. Consider Q^2_{n-1}. kvertices are dominated by k vertices of Q^1_{n-1}. Rest of $2^{n-1} - k$ vertices in Q^2_{n-1} must be dominated. Choose $l - 1$ vertices among vertices of Q^2_{n-1}so that weak convexity is maintained among $l - 1$ vertices. Suppose there are m vertices from k vertices in Q^1_{n-1}that are for domination in Q^1_{n-1}then these m vertices have private neighbors in Q^1_{n-1}itself. So private neighbors of these m vertices in Q^2_{n-1} must be chosen so that domination is not violated in Q^2_{n-1}.

Thus $m + x = l - 1$. Now $k - (l - 1)$ private neighbors in Q^2_{n-1} of k vertices are not chosen. Since $l < k$ and we choose $l-1$ vertices, there are atleast two vertices in $k - (l - 1)$.

Let $k -(l -1) = 2$. Let these vertices be u, v. Let private neighbors of u and v be u_1 and v_1 respectively in Q^1_{n-1}. u_1, v_1 are among k vertices. Since u,vare not among $l -1$ vertices of Q^2_{n-1}, u_1, v_1 do not contribute for domination in Q^1_{n-1}. Suppose u_2, v_2 are adjacent to both u_1 and v_1 then Suppose private neighbors of u and v form an edge in Q^1_{n-1}. Then single vertex that dominates u and v is either u or v. Therefore weak convexity is violated between u(v) and a vertex among k vertices which is adjacent to private neighbors of u(v) in Q^1_{n-1}. Thus a contradiction.

If private neighbors of u and v do not form an edge in Q^1_{n-1} and $N(u)\cap N(v) \neq \varphi$ then weak convexity is violated in Q^2_{n-1} which is a contradiction.

If private neighbors of u and v do not form an edge in Q^1_{n-1} and $N(u)\cap N(v) = \varphi$ then either u or v is required for domination in Q^2_{n-1}. Thus weak convexity is violated between u(v) and a vertex among k vertices which is adjacent to private neighbors of u(v) in Q^1_{n-1}. Thus a contradiction.

Therefore, $k - (l - 1)<2$. Hence minimum one vertex must be included in any one of the layers of Q_{n-1} for a weak convex dominating set in Q_n.

Case (ii)

None of $l - 1$ vertices are private neighbors of k vertices. Clearly weak convexity is violated between any vertex of $Q^1_{n-1}$and Q^2_{n-1}.

Case (iii)

Some of $l - 1$ vertices are private neighbors of k vertices. By Case (i) we get the result. Interchanging k and l we get the result for $k < l$.

Conclusion

In this paper we determined weak convex domination number for hypercube graphs. We also determined convex, weak convex, m - convex and l_1-convex numbers of star and hypercube graphs. Other domination parameters for hypercubes are under study in our group.

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