On p- Open Sets with Respect to an Ideal

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Abstract

In this, paper we introduce and investigate, the notion of α $_p-$*open, semi*$-I_p-$*open and pre*$-I_p-$*open sets via idealization by using p- local function and studied their some properties.*

Keywords: Semi open, Pre open, Alpha open, P–Local function.

Introduction

Ideal in topological space have been considered since 1930 by Kuratowski [1] and Vaidyanathaswamy [2]. After that ideal topology generalized in general topology by Jankovi and Hamleet [3]. In 2005 Hatir and Noir iintroduced the $\alpha-I-open\ set$, $semi-I-open\ set$, $pre-I-openset$ [4]. Finally in 2014 $\alpha-I_S-open$, semi $-I_S-open$, $pre-I_S-open$ sets are introuced by R.Shanthi and M.Rameshkumar [5]. In this paper we introduced the notion of $\alpha-I_p-open$, $semi-I_p-open$, $pre-I_p-open$ set and studied some properties of their.

Preliminaries

Let (X, τ) be topological space with no separation properties assumed. For a subset of topologicalspace (X, τ), **Cl (A)** and **Int (A)** denote the closure and interior of A in (X, τ) resp. An ideal I of topological space is collection of non-empty subset of X together with the following.

(i) $A\epsilon r$ and $B\subseteq A$ $implies$ $B\epsilon r$(ii) $A\in r$ and $B\in r$ $implies$ $AUB\in r$. The triplet forms. (X, τ, I) is called the ideal topological space where ı is topological space of X with an ideal I. Given a topological space (X, τ) with an ideal Ion X If P(x) is the set of all subset of X, a set operator $(.)^*$: $P(x)\mathbf{e}P(X)$, called a local function [5] of A with respect to τ and I is defined as follows: for $\boldsymbol{A\subseteq X, A^{*(I,r)}=\{x\in X/U\cap A\notin I\}}$ **for every** $\boldsymbol{U\epsilon r(x)\ wherer\ (x)} = \{\boldsymbol{U\epsilon r/x\epsilon U}\}$. Additionally, **cl*(A) = AUA*** defines kuratowski closure operator for a topology τ^* (I, τ), called the *- topology and finer than τ.

Definition 2.1

Let (X, τ) be a topological space. A subset A of X is said be a p-open set [6] if there exists anopen set U in X such that $U \subseteq A \subseteq int(Cl(A))$. The complement of p-open set is p–closed. The collection of all p–open sets in X is denoted by pO(X) is called the p-local function. The semi closure of A in (X,τ) is denoted by the intersection of all p-closed setcontaining A and is denoted by $pcl(A)$.

Definition 2.2

For $A \subseteq X, A^{(*,I,r)} = \{x \in X / U \cap A \notin I\}$, for every $U \in pO(X)$ $where$ $pO(X,x) = \{U \epsilon pO(X) / x \epsilon U\}$we write A_* $instead\ of$ $A^{(*,I,r)}.r^{*p}(I) = \{U \subseteq X : Cl^{*p}(X-U) = X-U\}$.The closure operator Cl^{*p} for a topology $r^{*p}(I)$ is defined as follows $Cl^{*p}(A) = AUA_*$ for a topology $r \subseteq r^*(I) \subseteq r^{*p}(I)$ and $Int^{*p}(A)$ denotes the interior of the set A in (X, r^{*p}, I).

Definition 2.3

A Subset of topological space X is said to be,

$Pre-open, if\ A \subseteq int\ (Cl(A))$

$Semi-open, if A \subseteq Cl\ (int(A))$

$\alpha-open, if\ A \subseteq int\ (Cl(int(A))$

Definition 2.4

A Subset of topological space X is said to be,

$\alpha-I-open, if\ A \subseteq int\ (Cl^*(int(A)))$

$Pre-I-open, if A \subseteq int\ (Cl^*(A))$

$Semi-I-open, if A \subseteq Cl^*\ (int(A))$

Lemma: For a subset of topological space, the following properties hold.

$pcl(A) = AUint(cl(A))$

$pcl(A) = int(cl(A)), if A\ is\ open$

Lemma: let A be a topological space and A,B be subsets of X. then following properties hold:

$if\ A \subseteq B\ then\ A_* \subseteq B_*$

$if\ U \in r\ then\ U \cap A_* \subseteq (U \cap A)_*$

$A_* = pCl(A_*) \subseteq pCl(A) and\ A_*\ is\ p-closed\ in\ X$

$(A_*)_* \subseteq A_*$

$(AUB)_* = A_*UB_*$

$if\ I = \{\varphi\}, then\ A_* = pCl(A)$

$\alpha-I_p-$ open, Semi$-$ I_p- open ,pre $-$ I_p- open

In this we define the $\alpha-I_p-open$ sets, $Pre-I_p-open$, $Semi-I_p-open$ and studied some properties of their.

Definition 3.1

A Subset of topological space X is said to be.

$\alpha -I_p-open, if\ A \subseteq int(Cl^{*p}(int(A)))$

$Pre - I_p - open, if\ A \subseteq int(Cl^{*p}(A)$

$Semi-I_p-open, if\ A \subseteq Cl^{*p}(int(A))$

Proposition 3.2

For a subset of an ideal topological space the following hold:
*Every $\alpha - I_p - open$ set is $\alpha - open$.

Proof

Let A be aα $-I_p-open\ set$.Thus,we have $A \subseteq int\ (Cl^{*p}(int(A)))=int(int(A)_*U\ int(A)) \subseteq int(pcl(int(A))Uint(A)) \subseteq int(Cl(int(A))Uint(A)) \subseteq int(Cl(int(A))$. A is an $\alpha -open$*Every
*Every $Semi -I_p-open$ set is $Semi-open$.

Proof

Let A be a $Semi -I_p-open\ set$. Thus,we have $A \subseteq Cl^{*p}(int(A)) = int(A)_*U\ int(A) \subseteq pcl(int(A)) Uint(A) \subseteq Cl(int(A))Uint(A)) \subseteq Cl(int(A)$. A is an $Semi-open$.

*Every$pre-I_p-open$setisP$re\ -open$.

Proof:

Let A be $pre\ -I_p-open\ set$. Thus,we have $A \subseteq int(Cl^{*p}(A)) = int((A)_*U(A)) \subseteq int(pcl(A)\ UA \subseteq int(Cl(A)UA) \subseteq int(Cl(A))$. A is an P$re\ -open$.

Remark 3.3

Converse of the above proposition need not be true as seen from the following example.

Example 3.4

Let X={a,b,c},τ= {φ,{a},{b},{a,b},{a,d},{abd},X} and I={φ,{b},{c},{bc}}. then the set A = {b,c}, B = {a,b,c,d}, C = {a}. A is $Semi-open$, but not $Semi-I_p-open$, B is P$re\ -open$, but not P$re\ -I_p-open$, C is $\alpha-open$, but not $\alpha-I_p-open$.

Proposition 3.5

Every open set of an ideal topological space is an $\alpha-I_p-open\ set$.

Proof:

Let A be a $Semi\ -I_p-open\ set$. Thus,we have $A = (int(A)) \subseteq int(int(A)_*Uint(A))=int(Cl^{*p}(int(A)))$.Then **A** is an $\alpha\ -I_p-open\ set$.

Remark 3.4

Converse of the above proposition 3.3 need not be true as seen from the following example.

Example 3.6

Let X = {a,b,c,d}, τ ={φ,{a},X} and I={φ,{b},{c},{b,c}}. Set A={a,c}, is $\alpha-I_a-open$, $but\ A \notin \tau$

Proposition 3.7

Every $\alpha\ -I_p-open\ set\ is\ both$ Semi$-I_p-open\ set\ and$ P$re\ -I_p-open\ set$.

Proof

The proof is obvious.

Remark 3.8

Converse of the above proposition 3.7 need not be true as seen from the following example.

Example 3.9

Let X = {a, b, c}, τ={φ, {a}, {b}, {a, b}, {a, d}, {a, b, d}, X} and I = {φ, {b}, {c}, {b, c}}. then the set A = {a} is a $Pre-I_p-open$, but not $\alpha-I_p-open$ and A is $Semi-open$, but not $\alpha-I_p-open$.

Proposition 3.10

For a subset of an ideal topological space the following hold:
Every $\alpha-I_p-open$ set is $\alpha-I-open$.
Every $Semi-I_p-open$ set is $semi-I-open$.
Every $Pre-I_p-open$ set is $pre-I$-$open$

Proof:

The proof is obvious.

Remark 3.11

Converse of the proposition 3.10 need not be true. DFFD

Proposition 3.12

Let (X, τ, I) be an ideal topological space and A an open subset of X. Then the following hold, if I={φ}, then

1. A is $\alpha - I_p - open$ set if and only if $A\ is\ a\ \alpha - open$.

Proof

If I= {φ}, A= plc(A) for any subset Ao f X and hence Cl*p(A) = A∗UA=pCl(A).By proposition 3.2. Every $\alpha-I_p-open\ set$ is an $\alpha-open\ set$. Conversely if A is $\alpha-open\ set$. Then $A\subseteq int\ (Cl(int(A))) = \alpha cl(int(A) = (Cl^{*p}(int(A)).\ hence\ A = int(A) \subseteq int\ (Cl^{*p}(int(A)))$. Therefore, A is $\alpha-I_p-open$. Thus, A is $\alpha-I_p-open$ set if and only if $\alpha-open$.

2. A is $Semi-I_p-open$ set if and only if $A\ is\ a\ Semi-open$.

Proof

If I= {φ}, A= pCl(A) for any subset A of X and hence Cl*p(A) = A∗UA=pCl(A). By proposition 3.2. Every $semi-I_p-open\ set$ is an $semi-open\ set$. Conversely is fA is $semi-open\ set$. Then $A \subseteq Cl(int(A)).\ Hence\ A = int(A) \subseteq int\ (Cl(int(A))) = \alpha cl(int(A) = (Cl^{*p}(int(A))$. Therefore, A is $\alpha-I_p-open$. Thus, A is $semi-I_p-open$ set if and only if $semi-open$.

3. A is $Pre-I_p-open$ set if and only if $A\ is\ a\ Pre-open$.

Proof

If I= {φ}, A= pCl(A) for any subset A of X and hence Cl*p(A) = A∗UA= pCl(A). By proposition 3.2. Every $Pre-I_p-open\ set$ is an $pre-open\ set$. Conversely if A is $Pre-open\ set$. Then $A \subseteq int(Cl(A)) = \alpha Cl(A) = Cl^{*p}(A).\ Hence\ A = int(A) \subseteq int(Cl^{*p}(A))$. Therefore, A is $Pre-I_p-open$. Thus, A is $pre-I_p-open$ set if and only if $Pre-open$.

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