k-Extensibility and Weakly k-Extensibility in Generalized Petersen Graphs

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Abstract

Let G be a simple graph. Let k be a positive integer. G is said to be k- extendable if every independent set of cardinality k is contained in a maximum independent set of G. A graph is weakly k- extendable if any non-maximal independent set of cardinality k is contained in a maximal independent set of G. Every k- extendable is weakly k- extendable but not the converse. Thus weakly k- extendable graph is a class of graphs wider than the class of k-extendable graphs. k-extendable and weakly k- extendable have been studied in [1, 2,3,4,6]. Characterization of graphs with $\beta_0(G) = (n - 3)$, $\beta_0(G) = (n - 2)$, and which is trivially extendable has been done in [5]. In this paper, we derive the some results on k-extensibility and weakly k-extensibility in generalized Petersen graphs. "Detection of emerging communities in a social network proves helpful to trace the growth of certain interests or interest groups. There are many community detection algorithms in the literature. However, they have the limitations of being too loose or they are not scalable – i.e., inextensible to large social networks. In this paper, we define a new property for the generalized Petersen Graphs namely k-Extensibility, which helps to find and visualize such communities."

Keywords: Extensibility in graphs, Weaklyk-extendable graphs, Generalized Petersen graphs, Mathematics Subject Classification: 05C69.

Introduction

Independence and domination play an important role in graph theory. Many papers have been published on these topics. There are graphs which have unique maximum independent set and in some graphs every independent set is maximum. Maximal independent sets which are not maximum play an important role. The minimum cardinality of a maximal independent set is called the independence domination number of G and is denoted by i(G). Clearly $\gamma(G) \leq i(G) \leq \beta_0(G)$, where $\gamma(G)$ is the domination number of G and β_0 is the independence number of G. There may be maximal independent sets of cardinality between i(G) and $\beta_0(G)$. In this paper, we check some results on k-extensibility and weakly k-extensibility for generalized Petersen graphs.

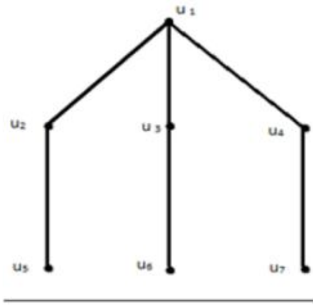
k-Extendable Graphs

Definition

Let G=(V,E) be a simple graph. Let k be a positive integer. G is said to be k-extendable if every independent set of cardinality k in G is contained in a maximum set of G.

Example:

Consider the graph



$S_1 = \{u_1, u_5, u_6, u_7\}$ is the unique maximum independent set of G, $\beta_0(G) = 4$. G is not 3-extendable, since $S_2 = \{u_2, u_3, u_4\}$ is an independent set of G cardinality 3 but is not contained in the unique maximum independent set of G. G is neither 1- extendable nor 2 extendable. Thus G is k-extendable for k=4.

Extensibility for Some Standard Graphs

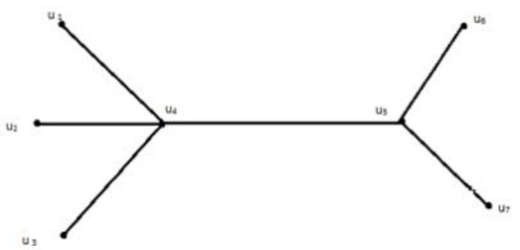
Complete graph k_n is k-extendable for k=1.
(i) Cycle C_{2n} is k-extendable k=1 and k= $\beta_0(G)$
(ii) Cycle C_{2n+1} is k -extendable for $1 \leq k \leq 2$ and k= $\beta_0(G)$
Star graph $k_{1,n}$ is k-extendable for $2 \leq k \leq 2\ \beta_0(G)$
(i) Path P_{2n} is k -extendable for k=1and k= $\beta_0(G)$
(ii) Path $P_{2n+1}$is k-extendable for k= $\beta_0(G)$
(i) Wheel W_{2n} is k-extendable for k=2 and k = $\beta_0(G)$, $n \geq 3$
(ii) Wheel W_{2n+1} is k-extendable for k= $\beta_0(G)$ only, $n \geq 2$
(i) Double Star $D_{r,s}$ is k-extendable for $(i + (G) + 1) \leq k \leq \beta_0(G)$ if r=s
(ii) Double Star $D_{r,s}$ is k-extendable for $(n - i(G) + 1) \leq k \leq \beta_0(G)$ if $r \neq s$
(i) $k_{m,n}$ is k-extendable for $1 \leq k \leq \beta_0(G)$ if m=n
(ii) $k_{m,n}$ is k-extendable for $(\min(m,n) + 1) \leq k \leq (\max(m,n))$ if $m \neq n$
Hajo's graph is k-extendable for k= $\beta_0(G)$
(i) Mycielskian graph $\mu(P_n)$ of the (P_n) is k-extendable for k=1,2 where n=2
(ii) Mycielskian graph $\mu(P_n)$ of the (P_n) is k-extendable for k= $\beta_0(G)$ where $n \geq 2$
Petersen graph G is k-extendable for k=1 and k= $\beta_0(G)$

Weakly k-Extendable Graphs

Definition

Let G be a graph. Let k be a positive integer, $1 \leq k \leq |V(G)|$. G is said to be weakly k - extendable if every non-maximal independent set of cardinality k of G is contained in a maximum independent set of G.

Example



Let G= $D_{3,2}$. Here $\beta_0(D_{3,2}) = 5$ and $\{u_1,u_2,u_3,u_6,u_7\}$is a unique maximum independent set of $D_{3,2}$, $\{u_5\}$,$\{u_5,u_1\}$, $\{u_5,u_1,u_2\}$ are non-maxima independent sets of cardinality 1,2,3 respectively, which are not in β_0 set of $D_{3,2}$. Hence $D_{3,2}$ is not weakly k-extendable for all, $1 \leq k \leq 3$.

Weakly k-Extendable for Some Standard Graphs

(i) C_{2n} is weakly k- extendable for k=1 and $(\beta_0(G) - 1)$, n=2,3,......
(ii) C_{2n+1} is weakly k-extendable for $1 \leq k \leq (\beta_0(G) - 1)$
$K_{1,n}$ is weakly k-extendable for k, $1 \leq k \leq (\beta_0(G) - 1)$
(i) P_{2n} is weakly k-extendable for k=1 and $k = 1 \leq k \leq (\beta_0(G) - 1)$
(ii) P_{2n+1} is weakly k-extendable for $k = (\beta_0(G) - 1)$ only.
(i) W_{2n} is weakly k-extendable k=1,2 and $k = (\beta_0(G) - 1)$, n=3,4,5….
(ii) W_{2n+1} is weakly k-extendable for k=1 and $k = (\beta_0(G) - 1)$, n=2,3,4,…..
(i) $D_{r,s}$ is not weakly k-extendable for $1 \leq k \leq (\beta_0(G) - 1)/2$ if r = s.
(ii) $D_{r,s}$ is not k-extendable for $1 \leq k \leq \max\{(s-1), (r-1)\}$ if $r \neq s$.
(i) $k_{m,n}$ is weakly k-extendable for $1 \leq k \leq (\beta_0(G) - 1)$ if m=n.
(ii) $k_{m,n}$ is not weakly k-extendable for k, $1 \leq k \leq \min\{(m-1), (n-1)\}$, if $m \neq n$
Petersen graph G is weakly k-extendable for k=1, $(\beta_0(G) - 1)$.

k-Extendable and Weakly k-Extendable in Generalized Petersen Graphs

Definition

Generalized Petersen Graphs P(m, n)

For each $n \geq 3$ and $0 < n < m$ P(m, n) denotes the generalized Petersen graph with vertex V(G) = $\{u_1, u_2. . . . u_m, v_1, v_2, . . , v_m\}$ and the edge set E(G) = $\{u_iu_{i+1(\mathrm{mod}\ m)}, u_iv_{i+k(\mathrm{mod}\ m)}\}$, $1 \leq i \leq n$.

Theorem

P(n, 3), n odd and $i(P(n,3)) = \beta_0(P(n,3))$ is k-extendable and weakly k-extendable for all k, for all k, $1 \leq k \leq (\beta_0(P(n,3) - 1))$

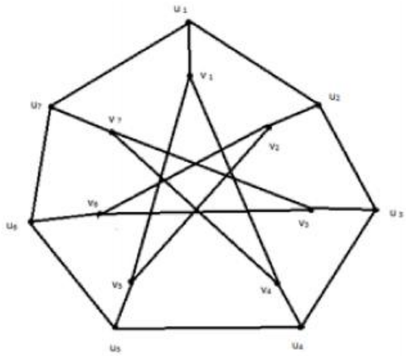
Proof

Let $V(P(n, 3))=\{u_1, u_2, ... , u_n, v_1, v_2, . . . v_n\}$$E(P(n, m))=\{u_iu_i+_1, v_iv_i+_3, u_iv_i\ (\mathrm{mod})\}$, where $1 \leq k \leq n$. $\beta_0(P(n,3) = n - 2$. The following are β_0-sets $\{u_1, v_2, u_3, v_4, ..., u_{n-2}, v_{n-2}\}$, $\{v_1, u_2, v_3, u_4,$

...., v_{n-2}}, {u_{n-1}, v_n, u_1, v_2, ..., u_{n-3}} and {v_{n-1}, u_n, v_1, u_2, ..., v_{n-3}}, {u_n, v_1, u_2, ..., u_{n-3}}, {v_1, u_2, v_2, ..., v_{n-3}}. Also i(P(n,3)) = β_0 (P(n,3)). Clearly any independent sets and non-maximal independent sets of cardinality 1 to β_0 (P(n,3)) - 1 is contained in a maximum independent set of P(n, 3), hence it k-extendable and weakly k-extendable for all k, $1 \le k \le (\beta_0$ (P(n,3) - 1)).

Example

Let G=P(7, 3)



The β_0-sets of G=P(7, 3) are

{u_1, u_3, u_5, v_2, v_4}, {u_2, u_4, u_6, v_1, v_3}, {u_1, u_4, u_6, v_3, v_5}, {u_1, u_4, u_6, v_2, v_7}, {u_1, u_3, u_5, v_4, v_6}, {u_1, u_3, u_5, v_4, v_6}, {u_1, u_3, u_5, v_2, v_7}, {u_2, u_4, u_6, v_3, v_5}, {u_2, u_4, u_6, v_5, v_7}, {u_2, u_4, u_6, v_1, v_7}, {u_2, u_5, u_7, v_1, v_3}, {u_2, u_5, u_7, v_3, v_4}, {u_2, u_5, u_7, v_4, v_6}, {u_2, u_5, u_7, v_1, v_6}, {u_1, u_3, u_6, v_2, v_4}, {u_1, u_3, u_6, v_4, v_5}, {u_1, u_3, u_6, v_5, v_7}, {u_2, u_4, u_7, v_1, v_3}, {u_2, u_4, u_7, $v_{3,}$ v_5}, {u_2, u_4, u_7, v_5, v_6}, {u_2, u_4, u_7, v_1, v_6}, {u_3, u_5, u_7, v_1, v_2},{u_3, u_5, u_7, v_2, v_4},{u_3, u_5, u_7, v_4, v_6}, {u_3, u_5, u_7, v_1, v_6}.

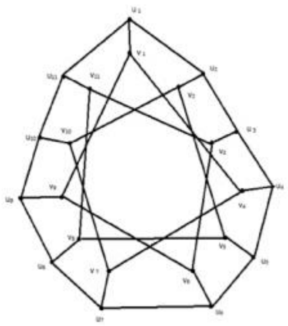
Here β_0(P(7, 3))=5=i(P(7, 4)). Also if we take any independent sets as well as non-maximal sets of cardinality 1,2,3,4 are all completely lies inside any one of the above β_0(P(7, 3)). Hence P(7, 3) is k-extendable for all k, $1 \le k \le (\beta_0$ (P(7,3) - 1)) and it is weakly k-extendable for all k, $1 \le k \le (\beta_0$ (P(7,3) - 1))

Remark

P(n, 3), n odd and i(p(n,3)$\neq \beta_0$ (p(n,3)) is not k-extendable for all k, $3 \le k \le (\beta_0$ (P(n,3)) - 2) and not weakly k-extendable for all k, $3 \le k \le (\beta_0$ (P(n,3)) - 3) .

Example

Let G=P(11, 3)



The β_0 - sets are
$\{u_1,u_3,u_5,u_7,u_9,v_2,v_4,v_6,v_8\}$, $\{u_1,u_3,u_5,u_7,u_9,v_4,v_6,v_8,v_{10}\}$, $\{u_1,u_3,u_5,u_7,u_9,v_2,v_4,v_6,v_{11}\}$, $\{u_1,u_3,u_5,u_7,u_9,v_4,v_6,v_{10},v_{11}\}$, $\{u_2,u_4,u_6,u_8,u_{10},v_1,v_3,v_5,v_7\}$, $\{u_2,u_4,u_6,u_8,u_{10},v_3,v_5,v_7,v_9\}$, $\{u_2,u_4,u_6,u_7,u_{10},v_5,v_7,v_9,v_{11}\}$, $\{u_2,u_4,u_6,u_8,u_{10},v_1,v_5,v_7,v_{11}\}$, $\{u_1,u_4,u_6,u_7,u_{10},v_2,v_7,v_9,v_{11}\}$, $\{u_1,u_4,u_6,u_7,u_{10},v_2,v_3,v_7,v_9\}$, $\{u_1,u_4,u_6,u_7,u_{10},v_5,v_7,v_9,v_{11}\}$, $\{u_1,u_4,u_6,u_7,u_{10},v_3,v_5,v_7,v_9\}$, $\{u_1,u_3,u_6,u_8,u_{10},v_2,v_4,v_9,v_{11}\}$, $\{u_1,u_3,u_6,u_8,u_{10},v_5,v_7,v_9,v_{11}\}$, $\{u_1,u_3,u_6,u_8,u_{10},v_2,v_7,v_9,v_{11}\}$, $\{u_1,u_3,u_6,u_8,u_{10},v_4,v_5,v_9,v_{11}\}$, $\{u_2,u_4,u_7,u_9,u_{11},v_1,v_3,v_5,v_{10}\}$, $\{u_2,u_4,u_7,u_9,u_{11},v_1,v_6,v_8,v_{10}\}$, $\{u_2,u_4,u_7,u_9,u_{11},v_1,v_5,v_6,v_{10}\}$, $\{u_2,u_4,u_7,u_9,u_{11},v_1,v_3,u8,v_{10}\}$, $\{u_2,u_5,u_7,u_9,u_{11},v_1,v_3,v8,v_{10}\}$, $\{u_2,u_5,u_7,u_9,u_{11},v_3,v_4,v_8,v_{10}\}$, $\{u_2,u_5,u_7,u_9,u_{11},v_4,v_6,v_8,v_{10}\}$, $\{u_2,u_5,u_7,u_9,u_{11},v_1,v_6,v_8,v_{10}\}$.

Here β_0(P(11, 3))=9 and i(P(11,3))=7. The maximal independents set is $\{u_1,u_4,u_7,u_{10},v_3,v_5,v_9\}$. P(11,3) is not k-extendable for k, 3≤ k ≤7. Since $\{u_1,u_4,u_7\}$, $\{u_1,u_4,u_7,u_{10}\}$, $\{u_1,u_4,u_7,u_{10},v_3\}$, $\{u_1,u_4,u_7,u_{10},v_3,v_5\}$, $\{u_1,u_4,u_7,u_{10},v_3,v_5,v_9\}$ are all independent sets of cardinality 3,4,5,6 and 7 which is not contained in any β_0(P(11, 3)). Clearly any non-maximal independent sets of cardinality k, 3≤ k ≤6 which is not contained in any β_0(P(11, 3)). Hence it is not weakly k-extendable for all k, 3≤ k ≤6.

Theorem

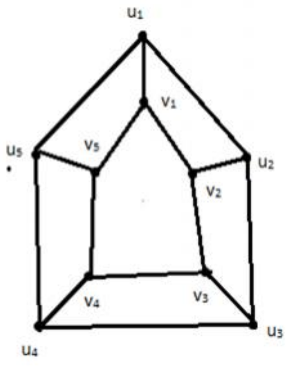
P(n, 1) n odd is k-extendable and weakly k-extendable for all k, 1 ≤ k ≤ (β_0 (P(n,1)) - 1)

Proof

Let V(P(n,1)= $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots v_n\}$
E(P(n, 1)= $\{u_iu_{i+1}, v_iv_{i+1}, u_iv_i$ (mod n)$\}$where 1 ≤ i ≤ n, β_0 (P(n,1)) - 1. The following are β_0- sets $\{u_1, v_2, u_3, v_4, \ldots, u_{n-2}, v_{n-1}\}$,$\{u_1, v_2, u_3, v_4, \ldots, u_{n-2}, v_{n-1}\}$, $\{u_1, v_2, u_3, v_4, \ldots, u_{n-3}, v_{n-2}\}$ and$\{v_nu_1, v_2, \ldots, v_{n-3}, u_{n-2}\}$. Clearly any independent sets and non-maximal independent sets of cardinality 1 to β_0(P(n,1)) - 1) is contained in a maximum independent set of P(n,1), hence it k-extendable and weakly k-extendable for all k, 1 ≤ k ≤ (β_0 (P(n,1)) - 1).

Example

Let G=P(5,1)



The β_0-sets are
$\{u_1, u_3, u_5, v_2\}$, $\{u_1, u_3, u_5,v_4\}$, $\{u_2, u_4, v_1, v_3\}$, $\{u_2, u_4, v_3, v_5\}$, $\{u_3, u_5, v_1, v_4\}$, $\{u_3, u_5, v_2, v_4\}$, $\{u_3, u_5, v_1, v_4\}$, $\{u_3, u_5, v_2, v_4\}$, $\{v_3, v_4, u_1, u_4\}$, $\{v_1, v_4, u_2, u_5\}$, $\{v_1, v_4, u_3, u_5\}$. Here β_0(P(5, 1))=4 and i(P(5, 1))=4. Also any independent sets as well as non-maximal independent sets of cardinality 1,2,3 which is contained in any one of the above β_0(P(5, 1)). Hence P(5, 1) is k-extendable and weakly k-extendable for k, 1 ≤ k ≤ 3.

Theorem

P(n, 2), n odd is k-extendable and weakly k-extendable for all k, $1 \le k \le (\beta_0(P(n,2)) - 1)$

Proof

$V(P(n, 2)) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots v_n\}$

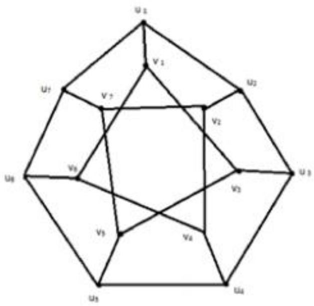
$E(P(n, 2)) = \{u_i u_{i+1}, v_i v_{i+2}, u_i v_i \text{ (mod n)}\}$ where $1 \le i \le n$, $\beta_0(P(n,2))=[4n/5]$.

The following are β_0- sets

$\{u_1, v_2, u_3, v_4, \ldots, v_{n-4}, u_{n-3,} u_{n-1}\}$, $\{v_1, u_2, v_3, u_4, \ldots, u_{n-4}, v_{n-3,} v_{n-1}\}$. Similar β_0-sets can be written starting with the remaining vertices of P(n, 2). Clearly any independent sets and non-maximal independent sets of cardinality 1 to $\beta_0(P(n,2))$-1 is contained in a maximum independent set of P(n, 2), hence it k-extendable and weakly k-extendable for all k,$1 \le k \le (\beta_0(P(n,2))-1)$.

Example

Let G=P(7,2)



The β_0-sets are

$\{u_1, u_3, u_5, v_2, v_6\}$, $\{u_1, u_3, u_5, v_4, v_7\}$, $\{u_1, u_3, u_5, v_6, v_7\}$, $\{u_2, u_4, u_6, v_1 v_5\}$,$\{u_2, u_4, u_6, v_3, v_7\}$, $\{u_2, u_4, u_6, v_1, v_7\}$, $\{u_1, u_4, u_6, v_2, v_3\}$, $\{u_1, u_4, u_6, v_3, v_7\}$, $\{u_1, u_4, u_6, v_2, v_5\}$, $\{u_2, u_5, u_7, v_1, v_4\}$, $\{u_2, u_5, u_7, v_3, v_4\}$, $\{u_2, u_5, u_7, v_3, v_6\}$, $\{u_1, u_3, u_6, v_2, v_5\}$, $\{u_1, u_3, u_6, v_4, v_5\}$, $\{u_1, u_3, u_6, v_4, v_7\}$, $\{u_2, u_4, u_7, v_1, v_5\}$,$\{u_2, u_4, u_7, v_3, v_6\}$, $\{u_2, u_4, u_7, v_5, v_6\}$, $\{u_3, u_6, v_1, v_2, v_5\}$, $\{u_4, u_6, v_1, v_2, v_5\}$, $\{u_3, u_7, v_1, v_2, v_5\}$, $\{u_4, u_7, v_1, v_2, v_5\}$, $\{u_1, u_4, v_2, v_3, v_6\}$, $\{u_1, u_5, v_2, v_3, v_6\}$, $\{u_4, u_7, v_2, v_3, v_6\}$, $\{u_5, u_7, v_2, v_3, v_6\}$, $\{u_1, u_5, v_3, v_4, v_7\}$, $\{u_1, u_6, v_3, v_4, v_7\}$, $\{u_2, u_5, v_3, v_4, v_7\}$, $\{u_2, u_6, v_3, v_4, v_7\}$, $\{u_2, u_6, v_1, v_4, v_5\}$, $\{u_2, u_7, v_1, v_4, v_5\}$, $\{u_3, u_6, v_1, v_4, v_5\}$, $\{u_3, u_7, v_1, v_4, v_5\}$, $\{u_1, u_3, v_2, v_5, v_6\}$, $\{u_1, u_4, v_2, v_5, v_6\}$, $\{u_3, u_7, v_2, v_5, v_6\}$, $\{u_4, u_7, v_2, v_5, v_6\}$, $\{u_1, u_4, v_3, v_6, v_7\}$, $\{u_1, u_5, v_3, v_6, v_7\}$, $\{u_2, u_4, v_3, v_6, v_7\}$, $\{u_2, u_5, v_3, v_6, v_7\}$, $\{u_2, u_5, v_1, v_4, v_7\}$, $\{u_2, u_6, v_1, v_4, v_7\}$, $\{u_3, u_5, v_1, v_4, v_7\}$, $\{u_3, u_6, v_1, v_4, v_7\}$

Here $\beta_0(P(7, 2))=5$ and $i(P(7, 2))=5$. Also any independent sets as well as non-maximal independent sets of cardinality 1,2,3,4 which is contained in any one of the above $\beta_0(P(7, 2))$. Hence P(7, 2) is k-extendable and weakly k-extendable for k, $1 \le k \le 4$.

Theorem

P(n, 5), n odd is not k-extendable for all k, $1 \le k \le (\beta_0(P(n,5))-5)$ and not weakly k-extendable for all k, $1 \le k \le (\beta_0(P(n,5))-6)$

Proof

Let $V(P(n, 5)) = \{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots v_n\}$.

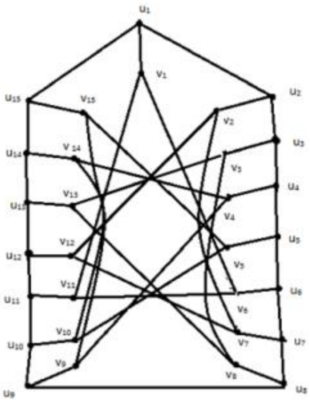
Let $E(P(n,5)) = \{u_i u_{i+1}, v_i v_{i+5}, u_i v_i, 1 \le i \le n \text{(mod n)}\}$

$\beta_0(P(n, 5))=n-3$. The following are β_0-sets $\{u_1, v_2, u_3, v_4, \ldots, v_{n-5}, u_{n-4,} u_{n-2}\}$, $\{v_1, u_2, v_3, u_4, \ldots,$

u_{n-5}, $v_{n-4,}v_{n-2}$}. Similar β_0-sets can be written starting with u_{n-1}, $v_{n-1,}u_n$; v_n; u_{n-3}, v_{n-3}.The maximal independent set of P(n,5) is {u_1, v_2, v_3, u_4, ..., v_{n-6}, $u_{n-5,}u_{n-2}$}. i(P(n, 5)=n-5. clearly any independent set of cardinality (n-5) which is not contained in any β_0(P(n,5)) and any non-maximal independent set of cardinality (n-6) is not contained in β_0(P(n, 5)). Hence P(n, 5) is not k-extendable for all k, 1 ≤ k ≤ (β_0 (P(n,5))-5) and not weakly k- extendable for all k, 1 ≤ k ≤ (β_0 (P(n,5))-6).

Example

Let G=P(15,5)



The β_0-sets are

{u_1,v_2,u_3,v_4,u_5,v_6,u_7,v_8,u_9, v_{10},u_{11},u_{13}}, {v_1,u_2,v_3,u_4,v_5,u_6,v_7,u_8,v_9, u_{10},v_{11},v_{13}}, {u_{15},v_{14},u_{13},v_{12},u_{10},v_9,u_8,v_7,u_6, v_5,v_4,u_3}, {u_1,u_3,u_5,u_9,u_{12},u_{14},v_2,v_4,v_6, v_8,v_{10}}

Here β_0(P(15, 5))=12 and i(P(15, 5))=10. Clearly any independent sets of cardinality k, 1 ≤ k ≤ 10 which is not contained in any β_0(P(15, 5)) and also any non maximal independent sets of cardinality k, 1 ≤ k ≤ 9 which is not contained in any β_0(P(15, 5)).

Application

Recently social networking has become an integral part of a digital person. He establishes social contact digitally, even among geographically distributed users. He uses the social network to (i) interact with each other, (ii) participate in online discussions, and (iii) even exchange their varying views forming social networks. Simply, in a mathematical framework, asocial network can be represented as a Graph, where-in the nodes represent the users (e.g. peoples, organizations, etc.) and edges or links denote the connections between the users.

A network contains a “Community Structure” if its possible to group the nodes of the network into particular sets of nodes, such that each set of nodes is connected internally densely among itself. In social networks, finding a community means finding a group of similar users who are like-minded people to people with same frequencies who interact on different entities like photos, comments, tags, stories or any other posts. Community detection is a significant task in Sociology, Biology and Computer Science disciplines where the system is often modeled as graphs. Hence detecting clusters or communities in large real-world graphs such as large social networks is a problem of considerable interest [1] [2]. Figure 1 shows a simple graph with three communities, enclosed by the dashed circles.

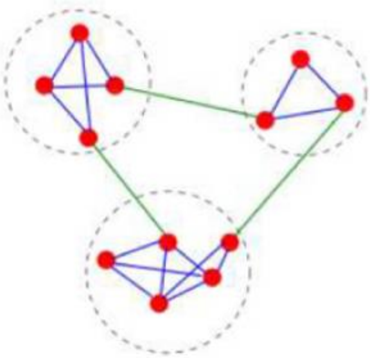


Figure 1 A Graph Showing Three Communities

In this paper, a generalized Petersen Graph based approach with k-extensibility property for community detection in social networks is proposed.

In a given Graph, the neighboring nodes are merged to form a single module, representing a community, and this merging process is repeated for all nodes in the graph. As a next iteration, the neighboring modules are joined to form super modules, indicating the community is growing. Initially, each node is assigned to its own module. Then, in a random and sequential order, every node is moved to its nearby neighboring module, and this process decreases the size of the network. If there is no further move which decreases the map equation of the network, the node remains in its original module. This procedure is repeated multiple times, every time in a new random and sequential order, until there is no more move that decreases the map equation. Thus the network is rebuilt, with the modules of the last level forming the nodes at this level, and, exactly as at the previous level, the nodes are joined into modules. While joining a node to a module, the presence of similarity with the existing nodes are checked out based on the k-extensibility property and forming of Petersen's graph. This algorithm results in a fairly good grouping of the network in a very short time and the clustering is also accurate.

The structure of generalized Petersen's graph is checked for its presence in each of the detected community. The k-extensibility property has to be checked for clustering of nodes and organizing them as belonging to a single community. It ensures the similarity property of a node to belong to a community, based on its activities like sharing, liking, checking-in, commenting, posting similar things in a social network. Nodes that are with similar interest will be identified accurately, using this k-extensibility property of the graph. To adapt to large-scale social network, a scalable parallel implementation of the proposed property on graphs has to be done, with powerful system features like multicores and cluster computing. The proposed approach can also be used to address various practical problems, such as check-in prediction and link prediction.

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