# New Analytical Solution of Stagnation Point Flow and Heat Transfer in a Porous Medium

# **OPEN ACCESS**

Volume: 8

Special Issue: 1

Month: May

Year: 2021

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#### Abstract

Impact Factor: 3.025

P-ISSN: 2321-788X

E-ISSN: 2582-0397

Citation:

Rajaram, M., et al. "New Analytical Solution of Stagnation Point Flow and Heat Transfer in a Porous Medium." *Shanlax International Journal of Arts, Science and Humanities*, vol. 8, no. S1, 2021, pp. 75–85.

# DOI:

https://doi. org/10.34293/sijash. v8iS4-May.4511 A new analytical method called q-homotopy analysis method is applied for solving nonlinear differential equations. In this paper, the mathematical study of stagnation points flow and heat transfer phenomena in a porous medium are discussed. The governing coupled nonlinear partial differential equations are converted into coupled nonlinear ordinary differential equations using similarity transformations and solved analytically for the values of Prandtl number Pr, Porous medium K, Casson fluid parameter  $\beta$ , Ratio parameter c using q-homotopy Analysis Method. The influence of the skin-friction coefficients for different parameters is discussed and presented in tabular form. The obtained q-homotopy analysis method solutions are compared with numerical results and it gives a remarkable accuracy.

Keywords: Similarity transformations, q-Homotopy analysis method, Numerical simulation, Skin-friction co-efficients and Local Nusselt number.

# Introduction

Stagnation point flow plays a vital rule in fluid mechanics. It has greatest importance for the prediction of Skin friction and local Nusselt numbers. The solution for two dimensional flow of a fluid near stagnation point was first proposed by Hiemeanz [1].

Many researchers [2] have been investigated about porous and non-porous boundary layers flow near the stagnation point of a stretching /shrinking sheet. Recently, Nandeppanavar et. al [3] discussed the stagnation point flow and heat transfer of Casson fluid over a stretching sheet. Nandeppanavar et. al has presented a numerical solution of Casson fluid flow using Runge Kutta Fourth order method [4-6].

The aim of this present work is to find the fluid flow over a stagnation point flow with uniform heat transfer [7,8]. The governing partial differential equations are converted into the non-linear ordinary differential equations using suitable similarity transformations [9,10]. The transformed ordinary differential equations are solved analytically. The solution for velocity profile and temperature distribution is obtained using the q-homotopy analysis method [11-15]. Estimation of Skin-friction Co-efficient and local Nusselt number is also presented in analytically. The effects of the pertinent parameters on the velocity components, temperature distribution are also analyzed.

## **Mathematical Formulation**

The boundary layer equations can be written as follows [4], u/x+v/y=0 (2.1)

$$\boldsymbol{u}\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{x}} + \boldsymbol{v}\frac{\partial\boldsymbol{u}}{\partial\boldsymbol{y}} = \boldsymbol{u}_{e}\frac{d\boldsymbol{u}_{e}}{d\boldsymbol{x}} + \boldsymbol{v}\left(1+\frac{1}{\beta}\right)\frac{\partial^{2}\boldsymbol{u}}{\partial\boldsymbol{y}^{2}} + \left(1+\frac{1}{\beta}\right)\frac{\boldsymbol{v}}{\boldsymbol{k}_{1}}\left(\boldsymbol{u}_{e}-\boldsymbol{u}\right)$$
(2.2)

$$\boldsymbol{u}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{x}} + \boldsymbol{v}\frac{\partial \boldsymbol{T}}{\partial \boldsymbol{y}} = \boldsymbol{\alpha}\frac{\partial^2 \boldsymbol{T}}{\partial \boldsymbol{y}^2}$$
(2.3)



where u and v are the velocity components of the fluid in x and y directions respectively.

v, and k\_1 are defined in [4].

The boundary conditions

$$u_w(x) = b_x, T = T_w \text{ at } y = 0, u_e(x) = a_x, T = T \text{ at } y$$
 (2.4)

Using Eqs.(2.2) and (2.3) we introduce the similarity transformations [4]. =(( $u_e(x)$ )/)^{(1/2)} y/x , =(  $xu_e$ )^{(1/2)} f( ), ( )=(T-T )/(T\_w-T ) (2.5)

where is the stream function. (1+1/)[Pr f^'' + K(1 - f^')] - f'^2 + 1 = 0 (2.6)

$$''+f'=0$$
 (2.7)

The boundary conditions

f' () = 
$$b/a = c$$
, f() = 0, () = 1 at =0 (2.8)

f' ( ) = 0, ( ) = 0 at = (2.9) where Pr=v/ is the Prandtl number and  $k=v/(k_1)$  is the porosity parameter.

The system of Eqs (2.6) and (2.9) is a coupled system of nonlinear differential equations

### Approximation Solution of the Flow and Temperature using q-HAM and HAM

The q-homotopy analysis method is a analytical technique for solving nonlinear problems. The basic concept of this method is given in [10]. Using the q-HAM, the approximate analytical expressions are given as follows:

 $f( ) = A (1-e^{-}) + B + h^{2}/n^{2} \{ ((c^{2} K)/(SPr^{2}))(5/2 - 3e^{-}) + (c^{3}/(S^{2}Pr^{2}))(4 - 6e^{-}) + (c/SPr) (1/SPr + K/Pr)(e^{-}) + 4e^{-} - 4) \}$  (3.1)

where A = c + 2hc/n - (2hKc/Prn) - ((4hc<sup>2</sup>)/SPrn) + (h<sup>2</sup> c)/n<sup>2</sup> - ((h<sup>2</sup> Kc)/(Prn<sup>2</sup>)) - ((h<sup>2</sup> c<sup>2</sup>)/(SPrn<sup>2</sup>)), B = (1/SPr+K/Pr)[K/Pr (h/n-h<sup>2</sup>/n<sup>2</sup>) + (n+1) h<sup>2</sup>]

 $()=C(1-e^{-}) + D(1-e^{-2}) + Ee^{-2} + F[e^{-}(+1)-1] + e^{-}(1-Ch^{2}/n^{2}) + (e^{-3}-1)4c^{2}h^{2}/3n^{2}) + ((h^{2} Kc)/(2Prn^{2})+1/n) (3.2)$ 

where C =  $(2h(1-2c))/n+h^2/n^2 (2c + Kc/Pr + (2c^2)/SPr - 4c^2 - 1), D = 2hc/n + h^2/n^2 (c/2 - 2c - c^2/SPr + 2c^2)$ 

 $E = h^2/n^2$  (c + Kc/Pr),  $F = h^2/n^2 (1/SPr + K/Pr)$ 

For n = 1, q-HAM can be converted into HAM. Eqs.(3.1) and (3.2) can be reduced into the following equations.

 $\begin{array}{l} f(\ ) = A \ (1 - e^{-}) + B \ (1/SPr + K/Pr) & + h^2 \left\{ ((c^2 \ K)/(SPr^2 \ ))(5/2 - 3e^{-}) + (c^3/(S^2Pr^2 \ ))(4 - 6e^{-}) + (c/SPr) \ (1/SPr + K/Pr) \ (e^{-} + 4e^{-} - 4) \right\} \end{array}$ 

where A = c + 2hc - (2hKc/Pr) - ((4hc<sup>2</sup>)/SPr) + h<sup>2</sup> c- ((h<sup>2</sup> Kc)/Pr) - ((h<sup>2</sup> c<sup>2</sup>)/SPr), B = (1/SPr + K/Pr) [K/Pr (h - h<sup>2</sup>) + 2 h<sup>2</sup>]

() = C (1 -  $e^{-}$ ) + D(1- $e^{-2}$ ) + E  $e^{-2}$  + F [ $e^{-}$  (+1) - 1] +  $e^{-}$  (1- Ch<sup>2</sup>) + ( $e^{-3}$  - 1) (4c<sup>2</sup> h<sup>2</sup>)/3 + ((h<sup>2</sup> Kc)/2Pr+1) (3.4)

where C=2h (1 - 2c) +  $h^2$  (2c + Kc/Pr + (2c^2)/SPr - 4c^2 - 1), D = 2hc +  $h^2$  (c/2 - 2c - c^2/SPr + 2c^2)

 $E = h^2 (c+Kc/Pr), F = h^2 (1/SPr + K/Pr)$ 

#### Analytical Expression for Skin Friction Co-efficient and Local Nusselt Number

The analytical expression for skin friction co-efficients as follows: f'' (0) = -A -  $(3h^2 c^2 k)/(n^2 SPr^2) - (6c^2)/(S^2 Pr^2) + 2c/SPr (1/SPr+K/Pr)$  (4.1)

For n = 1, q-HAM can be converted into HAM. Eqs.(4.1) can be reduced into the following equations.

$$f''(0) = -A - (3h^2 c^2 k)/SPr^2 - (6c^2)/(S^2 Pr^2) + 2c/SPr (1/SPr + K/Pr)$$
(4.2)

where  $A = c + 2hc - (2hKc/Pr) - ((4hc^2)/SPr) + h^2 c - ((h^2 Kc)/Pr) - ((h^2 c^2)/SPr)$ 

The analytical expression for local Nusselt number is '(0) = -{C+ 2[D - E - F] - 1 + ch<sup>2</sup> (1-4c)} (4.3) For n = 1, q-HAM can be converted into HAM. Eqs.(4.3) can be reduced into the following equations.

$$C(0) = -\{C + 2[D - E - F] - 1 + ch^{2}(1 - 4c)$$
(4.4)

where  $C = 2h(1 - 2c) + h^2(2c + Kc/Pr + (2c^2)/SPr - 4c^2 - 1), D = 2hc + h^2(c/2 - 2c - c^2/SPr + 2c^2)$ 

 $E = h^2 (c + Kc/Pr), F = h^2 (1/SPr+K/Pr)$ 

#### **Results and Discussion**

The velocity profiles and temperature distribution are discussed for various physical parameters , K, Pr, c in the graphs. Eqs. (2.6) and (2.9) solved by q-homotopy analysis method and homotopy analysis method. The obtained analytical results are compared with the numerical result in Figs. (2) - (11) for different values of parameters. It gives good agreement with the numerical result.

The velocity of the flow for different parameters are presented in the graphs Figs.2 - 5. From the Fig.2 exhibits that velocity profile f'() decreases with increases. In Fig.3 indicates that an increases K various values of the velocity profile is also increases. Fig.4 visualizes the parameter Prandtl number Pr on the velocity flow f'(). There is prominent decreases in the velocity and parameter Prandtl number Pr increases. Fig.5 illustrate that an increases in the ratio parameter c also increases the velocity profile. The temperature distribution of the difficult parameters are plotting in the graphs Figs.6 - 9. From Fig.6 depict to analyze the temperature for various values . It is noticed that the () increases with an increases in the parameter. In Fig.7 it is evident that the temperature with on the parameter K. It is clearly that the parameter K increases with temperature decreases. Fig.9 shows the influence of the temperature increases with ratio parameter c is also increases.

#### Conclusion

Analytical solution of velocity flow and heat transfer are obtained for all values of parameter using q-homotopy analysis method. The effects of casson fluid parameter and porosity parameter K on velocity flow and temperature are opposite. When the casson parameter increases, the skin friction co-efficients decreases. The q-homotopy analysis method, will be applicable for other strongly nonlinear problems.

f'()	n = 1	n = 3	n = 10
0	-2.65058	-2.09244	- 1.88867
2	0.06462	- 0.17209	- 0.22543
4	0.42740	0.08728	- 0.00038
6	0.47586	0.12231	0.03007
8	0.48234	0.12704	0.03418
10	0.48320	0.12768	0.03475

Table 1: The Values of Velocity Flow f' ( ) with h = 0.2

<b>Table 2: The Values of Temperature</b>	(	)	with $h = -$	0.	.1
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( )	n = 1	n = 10	n = 100
0	1.97433	1.09974	1.00999
2	0.64266	0.19062	0.14091
4	0.40553	0.06329	0.02288

6	0.37207	0.04598	0.00689
8	0.36749	0.04363	0.00473
10	0.36687	0.04332	0.00444

Table 3: Numerical values of skin-friction coefficients f''(0) Eqs.(4.1) and (4.2)and local Nusselt number - ^' (0) Eqs.(4.3) and (4.4) for the various values ofparameters , K, Pr, c, h.

			-				
				q-HAM		HA	AM
	K	Pr	с	n=	n=10		=1
				f"(0)	- '()	f"(0)	- '()
0.5	0.5	1.0	-1.8	1.92370	1.01957	3.15392	1.15733
1.0				1.96842	1.01954	3.20832	1.15402
1.5				1.99558	1.01952	3.00948	1.15200
				2.10675	1.01944	1.90688	1.14400
	0.5			1.92369	1.01957	3.15392	1.15733
	1.0			1.88645	1.01947	2.66912	1.14733
	1.5			1.84920	1.01937	2.18432	1.13733
		1.0		1.92369	1.01957	3.15392	1.15733
		1.5		1.90659	1.01963	2.95534	1.16289
		2.0		1.89808	1.01966	2.86028	1.16567
			-1.8	1.92369	1.01957	3.15392	1.15733
			-1.5	1.59072	1.01963	2.47167	1.16333
			-1.0	1.04684	1.01973	1.48445	1.17333



Figure 2: Variation of velocity profile for various values of



Figure 4: Variation of velocity profile for various values of Pr



Figure 3: Variation of velocity profile for various values of K



Figure 5: Variation of velocity profile for various values of c



Figure 6: Variation of temperature profile for various values of



Figure 8: Variation of temperature profile for various values of Pr



Figure 10: Variation of velocity profile for various values of n



Figure 7: Variation of temperature profile for various values of K



Figure 9: Variation of temperature profile for various values of c



Figure 11: Variation of temperature profile for various values of n

#### Nomenclature

a, b, c	Constants
f	Dimensionless stream function
f^'	Dimensionless velocity
k	Thermal conductivity
K	Permeability parameter

K <sub>1</sub>	Permeability of the porous medium
Т	Fluid temperature
T <sub>w</sub>	Surface temperature
Т	Ambient temperature
u, v	Velocity components along x and y directions respectively
х, у	Cartesian co-ordinates along the surface normal to it respectively

## **Greek Letters**

α	Thermal diffusivity
β	Non-newtonian/casson parameter
η	Similarity variable
υ	Kinematic viscosity
θ	Dimensionless temperature
ψ	Stream function
w	Condition at the surface
~	Condition away from the surface

# Appendix A

The q-Homotopy analysis method is used to solve the Eqs.(2.6) and (2.9) with the suitable initial guess [10].

$f_0() = c(1-e^{-})$	(A.1)

$$_{0} = e^{-1}$$
 (A.2)

consider Eqs (2.6) and (2.7) is

N[f(;q)] = 0 and N[(;q)] = 0

where  $q \quad [0,1/n]$ .

The auxiliary linear operator as L = / T with  $L(C_1) = 0$  where  $C_1$  is arbitrary constant.

We construct the so-called zero order deformation equation [10]  $(1-nq)L[f(;q) - f_0()] = qhH()N[(f(;q)](1-nq)L[(;q) - _0()] = qhH()N[((;q)])$  where h 0 and H() 0

L[f(;q)]=0 when f(;q)=0 L[(;q)]=0 when (;q)=0

Taking q = 0 $f(;0) = f_0(), (;0) = 0$ 

Also taking q = 1 / n, we get  $N[f(;1/n)] = 0 \Rightarrow f(;1/n) = f()$  N[ ( ;1/n)] =0  $\Rightarrow$  ( ;1/n)= ( )

Thus q [0,1/n ], solution of f( ;q) and ( ;q) varies from initial condition  $\rm f_{_0}$  ( ) to f( ) and  $_{_0}$  ( ) to ( ) [11,12]

$$f(\eta; q) = f(\eta; 0) + \sum_{m=1}^{\infty} f(\eta) q^{m}$$
  
where  $f_{m}(\cdot) = 1/m! / q^{m} f(\cdot; q) = 0$   
$$\theta(\eta; q) = \theta(\eta; 0) + \sum_{m=1}^{\infty} \theta(\eta) q^{m}$$

where  $_{m}\left( \right. )=1/m!$  /  $q^{m}$  ( ;q)=0

Hence

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) \left(\frac{1}{n}\right)^m$$

where  $f_m() = 1/m! / q^m f(;q) |_{(q=0)}$ 

$$heta(\eta) = heta_0(\eta) + \sum_{m=1}^{\infty} heta_m(\eta) \left(\frac{1}{n}\right)^m$$

where  $_{_{m}}\left( \ \right) =1/m!$  /  $q^{m}$  ( ;q)  $|_{_{\left( q=0\right) }}$ 

The m<sup>th</sup> order deformation equation as follows [13,14]

$$L[f_{m}( ) - _{m}f_{(m-1)}( )] = qhH( ) R_{m}[f_{(m-1)}( )]$$

where  $R_{m} [f_{(m-1)} ( )] = 1/(m-1)! \quad {}^{(m-1)}/ q^{(m-1)} N[f( ;q)] |_{(q=0)}$ 

$$L[ _{m} ( ) - _{m} _{(m-1)} ( )] = qhH( ) R_{m} [ _{(m-1)} ( )]$$

where  $R_m [ _{(m-1)} ( )] = 1/(m-1)! q_{(m-1)} N[ ( ;q) ]|_{(q=0)}$ 

$$f_{m}(\eta) = k_{m} f_{m-1}(\eta) + h \int [f_{m-1}^{''}(\eta) - (\frac{K}{p_{\gamma}}) f_{m-1}(\eta) + (\frac{1}{SP_{\gamma}}) \sum_{i=0}^{m-1} f_{m-1-i} f_{i}^{'}$$
  
$$- (\frac{1}{SP_{\gamma}}) \sum_{i=0}^{m-1} f_{m-1-i}^{'} f_{i}^{'} + (\frac{1}{SP_{\gamma}} + \frac{K}{P_{\gamma}})] d\eta + a$$
  
$$\theta_{m}(\eta) = k_{m} \theta_{m-1}(\eta) + h \int [\theta_{m-1}^{''}(\eta) + \sum_{i=0}^{m-1} f_{m-1-i}(\eta) \theta_{i}^{'}(\eta)] d\eta + a$$

where S=(1+1/)

$$\begin{aligned} \mathbf{k}_{m} &= \begin{cases} \mathbf{0}, \quad m \leq \mathbf{1} \\ \mathbf{n}, \quad otherwise \end{cases} \\ f_{1}(\eta) &= h \int [f_{0}^{\prime\prime\prime} - \binom{K}{P_{r}} f_{0}^{\prime} + \binom{2}{SP_{r}} (f_{0}f_{0}^{\prime\prime\prime}) - \binom{2}{SP_{r}} f_{0}^{\prime} f_{0}^{\prime\prime} + \binom{1}{SP_{r}} + \frac{K}{P_{r}} )] d\eta + a \\ f_{1}(\eta) &= h \left\{ c(1 - e^{-\eta}) + \binom{Kc}{P_{r}} (e^{-\eta} - 1) + \binom{2c^{2}}{SP_{r}} (e^{-\eta} - 1) + \binom{1}{SP_{r}} + \frac{K}{P_{r}} \right) \eta \right\} \\ f_{2}(\eta) &= nf_{1}(\eta) + h \int [f_{1}^{\prime\prime\prime\prime} - \binom{K}{P_{r}} f_{1}^{\prime\prime} + \binom{1}{SP_{r}} (f_{1}f_{0}^{\prime\prime\prime} + 2f_{0}f_{1}^{\prime\prime\prime}) \\ - \binom{1}{SP_{r}} (f_{1}^{\prime} f_{0}^{\prime} + 2f_{0}^{\prime} f_{1}^{\prime}) + \binom{1}{SP_{r}} + \frac{K}{P_{r}} ]] d\eta + b \\ f_{2}(\eta) &= nh \left\{ c(1 - e^{-\eta}) + \binom{Kc}{P_{r}} (e^{-\eta} - 1) + \binom{2c^{2}}{SP_{r}} (e^{-\eta} - 1) \right\} + \binom{1}{SP_{r}} + \frac{K}{P_{r}} (n + 1)h\eta \\ + h^{2} \left\{ c(1 - e^{-\eta}) + \binom{Kc}{P_{r}} (e^{-\eta} - 1) + \binom{c^{2}}{SP_{r}} (e^{-\eta} - 1) + \binom{c^{2}K}{SP_{r}^{2}} \binom{5}{2} - 3e^{-\eta} \right\} \\ + \binom{c^{3}}{S^{2}P_{r}^{2}} (4 - 6e^{-\eta}) + \binom{c}{SP_{r}} \binom{1}{SP_{r}} + \frac{K}{P_{r}} (e^{-\eta} \eta + 4e^{-\eta} - 4) - \binom{1}{SP_{r}} + \frac{K}{P_{r}} \binom{K}{P_{r}} \eta \right\}$$
(A.4)

Solving the Eqs.(A.1), (A.3) and (A.4), using the boundary conditions Eqs. (2.8) and (2.9) and substituting into Eq.(A.5) and we can obtain Eq.(3.1) in the text.

$$\begin{aligned} \theta_{1}(\eta) &= h \int \left[ \theta_{0}'' + 2f_{0}\theta_{0}' \right] d\eta + a \\ \theta_{2}(\eta) &= n\theta_{1}(\eta) + h \int \left[ \theta_{1}'' + f_{1}\theta_{0}' + 2f_{0}\theta_{1}' \right] d\eta + b \end{aligned}$$
(A.6)  

$$\begin{aligned} \theta_{2}(\eta) &= nh\{(1 - 2c)(1 - e^{-\eta}) + c(1 - e^{-2\eta})\} + h^{2}\{(e^{-\eta} - 1) + 2c(1 - e^{-\eta}) \\ + 2c(e^{-2\eta} - 1) + \left(\frac{c}{2}\right)(1 - e^{-2\eta}) + \left(\frac{Kc}{2Pr}\right)(e^{-2\eta} + 1) + \left(\frac{Kc}{Pr}\right)(1 - e^{-\eta}) \\ + \left(\frac{c^{2}}{SPr}\right)(e^{-2\eta} - 1) + \left(\frac{2c^{2}}{SPr}\right)(1 - e^{-\eta}) + \eta e^{-\eta}\left(\frac{1}{SPr} + \frac{K}{Pr}\right) \\ &- \left(\frac{1}{SPr}\right)(1 + e^{-\eta}) - \left(\frac{K}{Pr}\right)(1 + e^{-\eta}) \\ + 4c^{2}(e^{-\eta} - 1) + 2c^{2}(1 - e^{-2\eta}) + \left(\frac{4c^{2}}{3}\right)(e^{-3\eta} - 1) + c(e^{-2\eta} - e^{-\eta}) \end{aligned}$$
(A.7)

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 $\theta(\eta) = \theta_0(\eta) + \theta_1(\eta) + \theta_2(\eta) + \dots$  (A.8) Solving the Eqs.(A.2),(A.3) and (A.4), using the boundary conditions Eqs. (2.8) and (2.9) and substituting into Eq.(A.8) and we can obtain Eq.(3.2) in the text.

#### Appendix B Using MATLAB Program Simulation of Eqs. (2.6) and (2.9).

```
function sol = ex5
ex5init=bvpinit (linspace(0,1),[0 -1 1 -2.5 10]);
sol = bvp4c(@ex5ode,@ex5bc,ex5init);
end
function dydx = ex5ode(x,y)
A=3;
p=1;
K=0.5;
dydx = [y(2)
  y(3)
  (1/A*p)*(y(2)*y(2)-1)+(K/p)+K/p*y(2)
  y(5)
  -(y(1)*y(5))
  1;
end
function res = ex5bc(ya,yb)
c = -1.8;
res=[
ya(1)
ya(2)-c
yb(2)
ya(4)-1
yb(4)
];
end
```

# To be typed in the command window

solution=ex5; x=solution.x; y=solution.y; y2=solution.y(2,:); y4=solution.y(4,:);

Plot(x,y2,'g',x,y4,'b');

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