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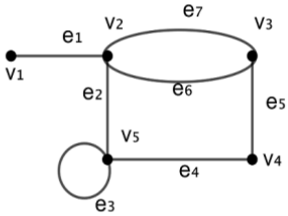
M. Radha
M.Phil. Scholar in Mathematics (PT), Reg. Number: 197215EP103
Mother Teresa Womens University, Kodaikanal

Dr. F. Silviya, M.Sc., M.Phil., Ph.D.,
Assistant Professor and Head, Department of Mathematics
Gonzaga College of Arts & Science for Women, Krishnagiri

Graphs in this chapter are simple. Terms here are used in the sense of Harary. The SML was focused as assignment of label to the vertices $x \in V$ with distinct elements f(x) from *1, 2, . . . , p* in such a way that when the edge $e = uv$ islabeled with $\frac{f(\alpha)+f(\beta)}{2}$iff(α) + f(β) is even and $\frac{f(\alpha)+f(\beta)+1}{2}$ if f(α) + f(β) is odd then the resulting edges get distinct labels from the set *{2, 3, . . . , p}*. In [2], we proved that if $n_1 \le n_2 < n_3$, the three star$K_{1,n_1} \cup K_{1,n_2} \cup K_{1,n_3}$is a SMGif $|n_2 - n_3| = 4 + n_1$ for n_1, n_2, n_3 are positive integers;also,$n_1 \le n_2 < n_3$, the three star$K_{n_1} \cup K_{n_2} \cup K_{n_3}$is not aSMG if $|n_2 - n_3| > 4 + n_1$ for n_1, n_2, n_3 are positive integers.; the graph $K_{1,n_1} \cup K_{1,n_1} \cup K_{1,n_2} \cup K_{1,n_3}$is a SMG if $|n_2 - n_3| = 4 + n_1$ for $n_1 = 2, 3, 4, . . .$; $n_2 = 2,3,4, . . .$; $n_3 = 2n_1 + n_2 + 4$ and $n_1 \le n_2 < n_3$; the graph$K_{1,n_1} \cup K_{1,n_1} \cup K_{1,n_2} \cup K_{1,n_3}$is not a SMG if $|n_2 - n_3| > 4 + n_1$ for $n_1 = 2, 3, 4, . . .$; $n_2 = 2,3,4, . . .$;$n_3 = 2n_1 + n_2 + 5$ and $n_1 \le n_2 < n_3$; the four star $K_{1,1} \cup K_{1,1} \cup K_{1,n_2} \cup K_{1,n_3}$ is a SMG if $|n_2 - n_3| = 7$ for$n_2 = 1,2,3, . . .$; $n_3 = n_2 + 7$ and $1 \le n_2 < n_3$ and the four star $K_{1,1} \cup K_{1,1} \cup K_{1,n_2} \cup K_{1,n_3}$ is not a SMG if $|n_2 - n_3| > 7$ for$n_2 = 1,2,3, . . .$; $n_3 \ge n_2 + 8$ and$1 \le n_2 < n_3$. In [3], the condition for a graph to be skolem mean is that $p \ge q + 1$.

Definition: Graph

A graph G = (V(G), E(G)), consists of two finite sets, V(G), the vertex set of the graph, often denoted by just V, which is non-empty sets of elementscalled vertices, E(G), the edges set of the graph, often denoted by just E, which is possibly an empty set of element called edges.



A graph G with five vertices and seven edges.

V(G) = {V_1, V_2, V_3, V_4, V_5,}

E(G) = {e_1, e_2, e_3, e_4, e_5, e_6, e_7,}

Definition: Empty Graph

An empty graph is graph with no edges.

In the graph empty graph with two vertices.

Definition

A graph G = (V, E) with p vertices and q edges is said to be a SMG if there exists a function f from the vertex set of G to {1, 2, . . . , p} such that the induced map f* from the edge set of G to {2, 3, . . . , p} defined by

$$f^*(e=\alpha\beta) = \begin{cases} \frac{f(\alpha)+f(\beta)}{2} & \text{if } f(\alpha)+f(\beta) \text{ is even} \\ \frac{f(\alpha)+f(\beta)+1}{2} & \text{if } f(\alpha)+f(\beta) \text{ is odd,} \end{cases}$$

the resulting edges get distinct labels from the set {2, 3, . . . , p}.

Some Results on Skolem Mean Graphs

In this chapter, we prove that the three stars K1,ℓ ∪ K1,p ∪ K1,q is a skolem mean graph if and only if | p–q | ≤ 4 + ℓ where ℓ = 1,2,3,…. And the four stars K1,ℓ ∪ K1,ℓ ∪ K1,p ∪ K1,q is a skolem mean graph if and only if | p–q | ≤ 4 + 2ℓ where ℓ = 2,3,4… . Also, we prove that the five stars K1,ℓ ∪ k1,ℓ ∪ k1,ℓ ∪ k1,p ∪ k1,q is a skolem mean graph if and only if | p–q | ≤ 4 + 3ℓ where ℓ =2,3,4,… . Finally we give the conjecture that the t stars t (k1,ℓ) ∪ k1,p ∪ k1,q is a skolem mean graph if and only if | p–q | ≤ 4 + tℓ where t =1,2,3,4… .

Theorem

K1,ℓ ∪ K1,p∪ K1,q is a skolem mean graph if | p–q | ≤ 4+ℓ where ℓ = 1, 2, 3, …. .

PROOF : Consider the graph K1,ℓ ∪ K1,p ∪ K1,q = K1, ℓ ∪ K1, (ℓ, ℓ +1 , ℓ +2 , ℓ +3 , ….) ∪ K1, (2ℓ +4 , 2ℓ +5 , 2ℓ +6 , …) where p = ℓ, ℓ +1, ℓ +2, ℓ +3, …. , q = 2ℓ +4, 2ℓ +5, 2ℓ +6,… and ℓ = 1, 2, 3, … . K1,ℓ ∪ K1,p ∪ K1,q = K1,ℓ ∪ K1, ℓ +t-1 ∪ K1, 2ℓ + t + 3 where ℓ = 1, 2, 3, … and t = 1, 2, 3, …. .

Case 1: Let ℓ = t = m.

Consider the graph K1,ℓ ∪ K1,p ∪ K1,q = K1,m ∪ K1, 2m-1 ∪ K1, 3m+3 . let {u},

{ ui:1 ≤ i ≤ m }, { v }, { vj:1 ≤ j ≤ 2m -1 } and { w }, { wk:1 ≤ k ≤ 3m + 3 } be the vertices of K1,m , K1,2m – 1 and K1,3m + 3 respectively. Then K1,m ∪ K1,2m–1 ∪ K1,3m + 3 has 6m + 5 vertices and 6m + 2 edges.

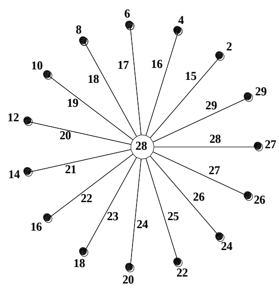
Define f: V (K1,m ∪ K1,2m–1 ∪ K1,3m+3) → {1, 2, 3, ..., 6m+5} by f (w) = 6m + 4, f (wk) = 2k, 1 ≤ k ≤ 3m + 1 and f (w3m+2) = 6m + 3, f (w3m+3) = 6m + 5. f (v) = 3, f (vj) = 2m + 2j + 3, 1 ≤ j ≤ 2m – 1 and f (u) = 1, f(ui) = m + 2i –1, 1 ≤ i ≤ m. The edge label of wwk is 3m + k + 2, 1 ≤ k ≤ 3m + 1, 6m + 4 and 6m + 5. The edge label of vvj is m + j + 3, 1 ≤ j ≤ 2m –1 and the edge label of uui is $\frac{m+2i}{2}$, 1 ≤ i ≤ m.

Hence the induced edge labels are 6m+2 distinct edges.

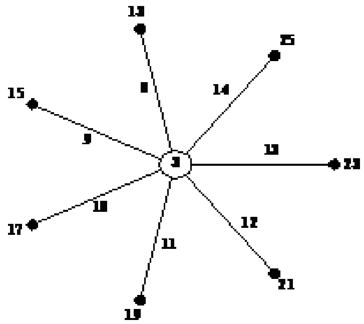
The Skolem mean labeling of K1, m ∪ K1, 2m–1 ∪ K1, 3m+3 are illustrated in Fig.2.0, Fig.2.1 and Fig.2.2 respectively.

Consider the graph G = K1,4 ∪ K1,7 ∪ K1,15 where m = 4.

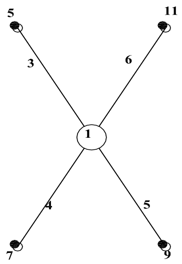
Then | v | = p = 29 and | E | = q = 26.



$K_{1,15}$



$K_{1,7}$



$K_{1,4}$

Therefore, all the edge labels are distinct in the graph.

Therefore, the graph G = $K_{1,4} \cup K_{1,7} \cup K_{1,15}$ is a skolem mean graph.

Hence the graph $K_{1,m} \cup K_{1,2m-1} \cup K_{1,3m+3}$ is a skolem mean graph.

Case 2: let ℓ = t = m+1.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+6}$. let {u}, { u_i:1 ≤ i ≤ m +1 }, { v },{ v_j:1 ≤ j ≤ 2m + 1 } and {w}, {w_k: 1 ≤ k ≤ 3m +6 } be the vertices of $K_{1,m+1}$, $K_{1,2m+1}$ and $K_{1,3m+6}$ respectively.Then $K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+6}$ has 6m + 11 vertices and 6m +8 edges.

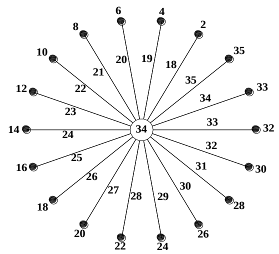
Definef: V($K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+6}$) → { 1, 2, 3, ..., 6m + 11 } by f(w) = 6m + 10, f(w_k) = 2k, 1 ≤ k ≤ 3m + 4 and f(w_{3m+5}) = 6m + 9, f(w_{3m+6}) = 6m+11. f(v) = 3, f(v_j) = 2m + 2j + 5, 1 ≤ j ≤ 2m +1 and f(u) = 1, f(u_i) = m + 2i –1, 1 ≤ i ≤ m +1. The edge label of ww_k is 3m +k +5,1 ≤ k ≤ 3m+4, 6m + 10 and 6m + 11. The edge label of vv_j is m + j+4, 1 ≤ j ≤ 2m+ 1 and the edge label of uu_i is $\frac{m+2i}{2}$, 1 ≤ i ≤ m + 1.

Hence the induced edge labels are 6m + 8 distinct edges.

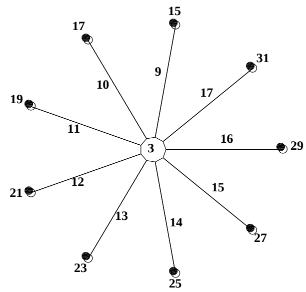
The Skolem mean labeling of $K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+6}$ are illustrated in Fig.2.3, Fig.2.4 and Fig.2.5 respectively.

Consider the graph G = $K_{1,5} \cup K_{1,9} \cup K_{1,18}$ where m = 4.

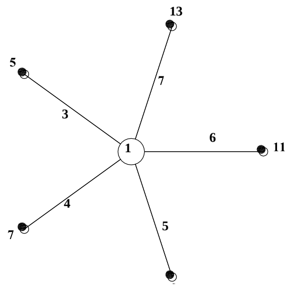
Then | V | = p = 35 and | E | = q = 32.



$K_{1,18}$



$K_{1,9}$



$K_{1,5}$

Therefore, all the edge labels are distinct in the graph.

Therefore, the graph $G = K_{1,5} \cup K_{1,9} \cup K_{1,18}$ is a skolem mean graph.

Hence the graph $K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+6}$ is a skolem mean graph.

Case 3: let $\ell = t = m + 2$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+9}$.let $\{u\}$, $\{u_i: 1 \le i \le m + 2\}$, $\{v\}$, $\{v_j: 1 \le j \le 2m + 3\}$ and $\{w\}$, $\{w_k: 1 \le k \le 3m+9\}$ be the vertices of $K_{1,m+2}$, $K_{1,2m+3}$ and $K_{1,3m+9}$ respectively. Then $K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+9}$ has 6m+17 vertices and 6m + 14 edges.

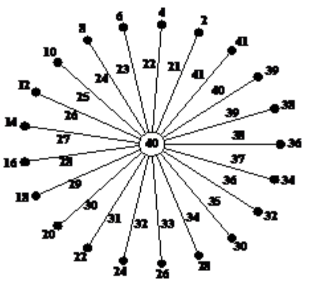
Define $f: V(K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+9}) \to \{1, 2, 3, \ldots, 6m+17\}$ by $f(w) = 6m + 16$, $f(w_k) = 2k$, $1 \le k \le 3m + 7$ and $f(w_{3m+8}) = 6m + 15$, $f(w_{3m+9}) = 6m + 17$. $f(v) = 3$, $f(v_j) = 2m + 2j + 7$, $1 \le j \le 2m + 3$ and $f(u) = 1$, $f(u_i) = m + 2i - 1$, $1 \le i \le m + 2$. The edge label of ww_k is $3m + k + 8$, $1 \le k \le 3m + 7$, $6m + 16$ and $6m + 17$. The edge label of vv_j is $m + j + 5$, $1 \le j \le 2m + 3$ and the edge label of uu_i is $\frac{m+2i}{2}$, $1 \le i \le m + 2$.

Hence the induced edge labels are 6m + 14 distinct edges.

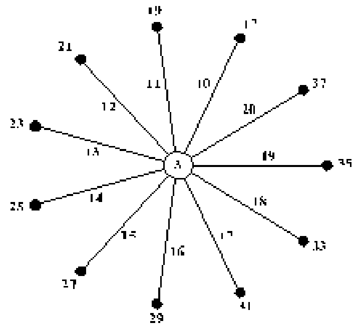
The Skolem mean labeling of $K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+9}$ are illustrated in Fig.3.6, Fig.3.7 and Fig.3.8 respectively.

Consider the graph $G = K_{1,6} \cup K_{1,11} \cup K_{1,21}$ where m = 4.

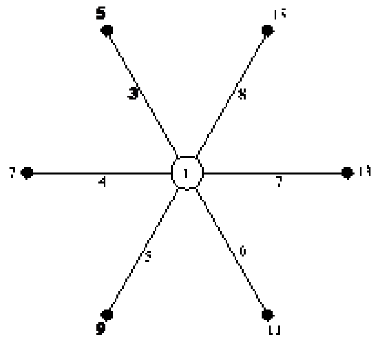
Then $|v| = p = 41$ and $|E| = q = 38$.



$K_{1,21}$



K1,11



K1,6

Therefore, all the edge labels are distinct in the graph.Therefore, the graph $G = K_{1,6} \cup K_{1,11} \cup K_{1,21}$ is a skolem mean graph.Hence the graph $K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+9}$ is a skolem mean graph.

Case 4: let $\ell = t = m + 3$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+12}$. let $\{u\}$, $\{u_i: 1 \le i \le m + 3\}$, $\{v\}$, $\{v_j: 1 \le j \le 2m + 5\}$ and $\{w\}$, $\{w_k: 1 \le k \le 3m + 12\}$ be the vertices of $K_{1,m+3}$, $K_{1,2m+5}$ and $K_{1,3m+12}$ respectively.Then $K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+12}$ has 6m+23 vertices and 6m+20 edges.

Define $f: V(K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+12}) \to \{1, 2, 3, \ldots, 6m + 23\}$ by $f(w) = 6m + 22$, $f(w_k) = 2k$, $1 \le k \le 3m + 10$ and $f(w_{3m+11}) = 6m + 21$, $f(w_{3m+12}) = 6m + 23$. $f(v) = 3$, $f(v_j) = 2m + 2j + 9$, $1 \le j \le 2m + 5$ and $f(u) = 1$, $f(u_i) = m + 2i - 1$, $1 \le i \le m + 3$. The edge label of ww_k is $3m + k + 11$, $1 \le k \le 3m + 10$, $6m + 22$ and $6m + 23$. The edge label of vv_j

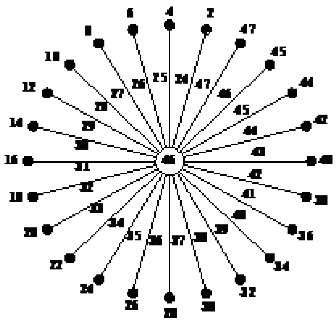
is $m + j + 6$, $1 \leq j \leq 2m + 5$ and the edge label of uu_i is $\frac{m+2i}{2}$, $1 \leq i \leq m + 3$.

Hence the induced edge labels are 6m+20 distinct edges.

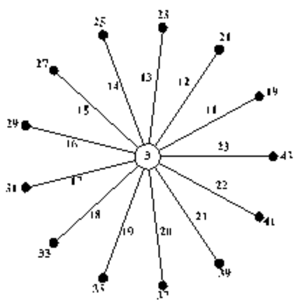
The Skolem mean labeling of $K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+12}$ are illustrated in Fig.3.9, Fig.3.10 and Fig.3.11 respectively.

Consider the graph $G = K_{1,7} \cup K_{1,13} \cup K_{1,24}$ where m = 4.

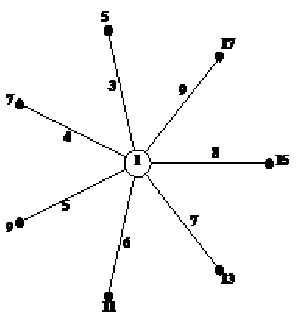
Then $| v | = p = 47$ and $| E | = q = 44$.



$K_{1,24}$



$K_{1,13}$



$K_{1,7}$

Therefore, all the edge labels are distinct in the graph. Therefore, the graph $G = K_{1,7} \cup K_{1,13} \cup K_{1,24}$ is a skolem mean graph. Hence the graph $K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+12}$ is a skolem mean graph.

Case 5: let $\ell = t = m + r$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+r} \cup K_{1,2m+2r-1} \cup K_{1,3m+3r+3}$ Where r = 0, 1, 2, 3,... let {u} , $\{u_i: 1 \leq i \leq m+r\}$, {v}, $\{v_j: 1 \leq j \leq 2m + 2r - 1\}$ and { w }, $\{ w_k: 1 \leq k \leq 3m + 3r + 3 \}$ be the vertices of $K_{1,m+r}$, $K_{1,2m+2r-1}$ and $K_{1,3m+3r+3}$ respectively. Then $K_{1,m+r} \cup K_{1,2m+2r-1} \cup K_{1,3m+3r+3}$ has 6m + 6r + 5 vertices and 6m + 6r + 2 edges.

Definef: $V (K_{1,m+r} \cup K_{1,2m+2r-1} \cup K_{1,3m+3r+3}) \rightarrow \{ 1, 2, 3, ..., 6m + 6r + 5 \}$ by $f (w) = 6m + 6r + 4$, $f (w_k) = 2k$, $1 \leq k \leq 3m + 3r + 1$ and $f (w_{3m+3r+2}) = 6m + 6r + 3$, $f (w_{3m+3r+3}) = 6m + 6r + 5$ $f (v) = 3$, $f (v_j) = 2m + 2j + 2r + 3$, $1 \leq j \leq 2m + 2r - 1$ and $f (u) = 1$, $f (u_i) = m + 2i - 1$, $1 \leq i \leq m + r$. The edge label of ww_k is $3m + 3r + k + 2$, $1 \leq k \leq 3m + 3r + 1$, $6m + 6r + 4$ and $6m + 6r + 5$. The edge label of vv_j is $m + j + r + 3$, $1 \leq j \leq 2m + 2r - 1$ and the edge label of uu_i is $\frac{m+2i}{2}$, $1 \leq i \leq m + r$. Hence the induced edge labels are 6m + 6r + 2 distinct edges. Conversely, suppose that $K_{1,\ell} \cup K_{1,p} \cup K_{1,q}$ is a skolem mean graph if $| p-q | > 4 + \ell$ Where $\ell = 1, 2, 3, \dots$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,\ell} \cup K_{1,(\ell, \ell+1, \ell+2, \ell+3, ...)} \cup K_{1,(2\ell+5, 2\ell+6, 2\ell+7,...)}$ where $p = \ell, \ell+1, \ell+2, \ell+3, \dots$,$q = 2\ell+5, 2\ell+6, 2\ell+7, ...$ and $\ell = 1, 2, 3, \dots$. $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,\ell} \cup K_{1,\ell+t-1} \cup K_{1,2\ell+t+4}$ where $\ell = 1, 2, 3, ...$ and $t = 1, 2, 3 ...$.

Case 6: let $\ell = t = m$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m} \cup K_{1,2m-1} \cup K_{1,3m+4}$. let { u }, $\{ u_i: 1 \leq i \leq m \}$, { v }, $\{ v_j: 1 \leq j \leq 2m - 1 \}$ and { w }, $\{ w_k: 1 \leq k \leq 3m + 4 \}$ be the vertices of $K_{1,m}$, $K_{1,2m-1}$ and $K_{1,3m+4}$ respectively. Then $K_{1,m} \cup K_{1,2m-1} \cup K_{1,3m+4}$ has 6m + 6 vertices and 6m + 3 edges.

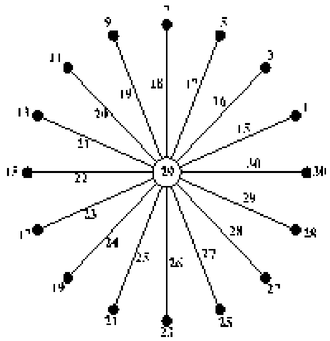
Define f: $V (K_{1,m} \cup K_{1,2m-1} \cup K_{1,3m+4}) \rightarrow \{1, 2, 3, ..., 6m+6\}$ by $f (w) = 6m + 5$, $f(w_k) = 2k-1$, $1 \leq k \leq 3m + 2$ and $f(w_{3m+3}) = 6m + 4$, $f (w_{3m+4}) = 6m + 6$. $f (v) = 4$, $f (v_j) = 2m + 2j + 4$, $1 \leq j \leq 2m - 1$ and $f (u) = 2$, $f (u_i) = m + 2i$,$1 \leq i \leq m$. The edge label of ww_k is $3m + k + 2$, $1 \leq k \leq 3m + 2$,$6m + 5$ and 6m

+ 6. The edge label of vv_j is $m + j + 4$, $1 \le j \le 2m - 1$ and the edge label of uu_i is $\frac{m+2i+2}{2}$, $1 \le i \le m$.

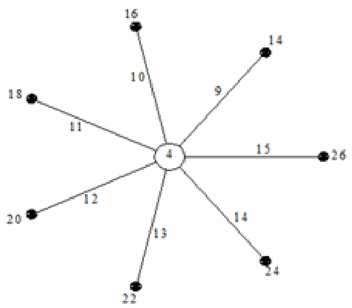
Hence the induced edge labels 6m + 3 are not receiving distinct edges.The Skolem mean labeling of $K_{1,m} \cup K_{1,2m-1} \cup K_{1,3m+4}$ are illustrated in Fig.2.12, Fig.2.13 and Fig.2.14 respectively.

Consider the graph $G = K_{1,4} \cup K_{1,7} \cup K_{1,16}$ where m = 4.

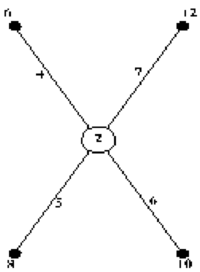
Then | V | = p= 30 and | E | = q = 27.



$K_{1,16}$



$K_{1,7}$



$K_{1,4}$

Therefore, the edge label of (29,1) is 15 in $K_{1,16}$ and the edge label of (4,26) is 15 in $K_{1,7}$.

Therefore, the two edge labels are same in the graph.

Therefore, the edge labels are not distinct in the graph.

Therefore, the graph $G = K_{1,4} \cup K_{1,7} \cup K_{1,16}$ is not a skolem mean graph.

Hence the graph $K_{1,m} \cup K_{1,2m-1} \cup K_{1,3m+4}$ is not a skolem mean graph.

Case 7: let $\ell = t = m+1$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+7}$. let { u }, { u_i: $1 \le i \le m + 1$ }, {v}, { v_j: $1 \le j \le 2m + 1$ } and {w}, {w_k: $1 \le k \le 3m + 7$ } be the vertices of $K_{1,\ m+1}$, $K_{1,\ 2m+1}$ and $K_{1,\ 3m+7}$ respectively.Then $K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+7}$ has 6m + 12 vertices and 6m + 9 edges.

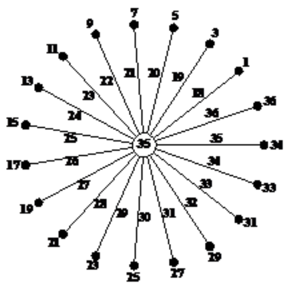
Definef: V ($K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+7}$) → { 1, 2, 3, ..., 6m + 12 } byf (w) = 6m + 11, f (w_k) = 2k – 1, $1 \le k \le 3m + 5$ and f (w_{3m+6}) = 6m + 10,f (w_{3m+7}) = 6m + 12.f (v) = 4, f (v_j) = 2m + 2j + 6, $1 \le j \le 2m + 1$ andf (u) = 2, f (u_i) = m + 2i, $1 \le i \le m + 1$. The edge label of ww_k is 3m + k + 5, $1 \le k \le 3m + 5$, 6m + 11 and 6m + 12. The edge label of vv_j is m + j + 5,$1 \le j \le 2m + 1$ and the edge label of uu_i is $\frac{m+2i+2}{2}$, $1 \le i \le m + 1$.

Hence the induced edge labels 6m+9 are not receiving distinct edges.

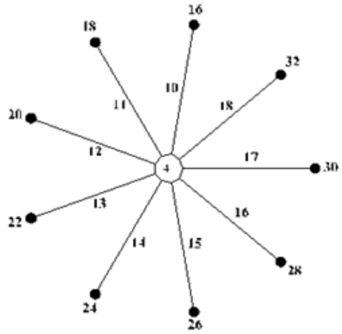
The Skolem mean labeling of $K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+7}$ are illustrated in Fig.2.15, Fig.2.16 and Fig.2.17 respectively.

Consider the graph $G = K_{1,5} \cup K_{1,9} \cup K_{1,19}$ where m = 4.

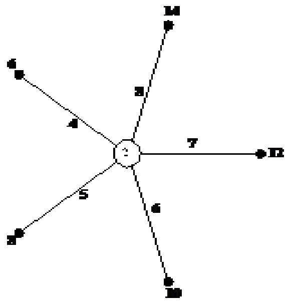
Then | V | = p = 36 and | E | = q = 33.



$K_{1,19}$



K1,9



$K_{1,5}$

Therefore, the edge label of (35,1) is 18 in $K_{1,19}$ and the edge label of (4,32) is 18 in $K_{1,9}$.

Therefore, the two edge labels are same in the graph.

Therefore, the edge labels are not distinct in the graph.

Therefore, the graph $G = K_{1,5} \cup K_{1,9} \cup K_{1,19}$ is not a skolem mean graph.

Hence the graph $K_{1,m+1} \cup K_{1,2m+1} \cup K_{1,3m+7}$ is not a skolem mean graph.

Case 8: let $\ell = t = m + 2$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+10}$. let { u }, $\{u_i: 1 \le i \le m+2\}$, {v}, $\{ v_j: 1 \le j \le 2m+3 \}$ and {w}, $\{ w_k: 1 \le k \le 3m+10 \}$ be the vertices of $K_{1,m+2}$, $K_{1,2m+3}$ and $K_{1,3m+10}$ respectively.Then $K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+10}$ has 6m + 18 vertices and 6m + 15 edges.

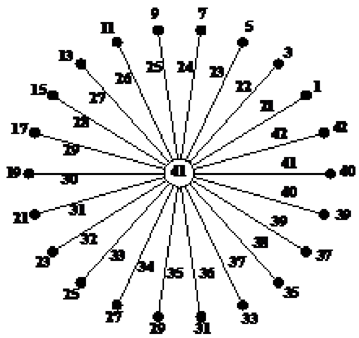
Definef: $V (K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+10}) \rightarrow \{ 1, 2, 3, ..., 6m + 18 \}$ by$f (w) = 6m + 17$, $f (w_k) = 2k - 1$, $1 \le k \le 3m + 8$ and $f (w_{3m+9}) = 6m + 16$,$f (w_{3m+10}) = 6m + 18$. $f (v) = 4$, $f(v_j) = 2m +2j + 8$, $1 \le j \le 2m + 3$ and $f (u) = 2$, $f (u_i) = m + 2i$, $1 \le i \le m + 2$. The edge label of ww_k is $3m + k + 8$, $1 \le k \le 3m + 8$, $6m + 17$ and $6m + 18$. The edge label of vv_j is $m + j + 6$, $1 \le j \le 2m + 3$ and the edge label of uu_i is $\frac{m+2i+2}{2}$, $1 \le i \le m + 2$.

Hence the induced edge labels 6m + 15 are not receiving distinct edges.

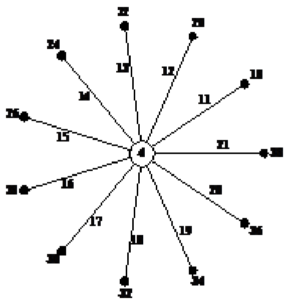
The Skolem mean labeling of $K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+10}$ are illustrated in Fig.2.18, Fig.2.19 and Fig.2.20 respectively.

Consider the graph $G = K_{1,6} \cup K_{1,11} \cup K_{1,22}$ where m = 4.

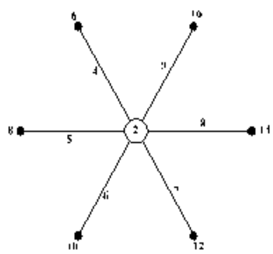
Then | v | = p = 42 and | E | = q = 39.



$K_{1,22}$



K1,11



$K_{1,6}$

Therefore, the edge label of (41,1) is 21 in $K_{1,22}$ and the edge label of (4,38) is 21 in $K_{1,11}$.

Therefore, the two edge labels are same in the graph.

Therefore, the edge labels are not distinct in the graph.

Therefore, the graph $G = K_{1,6} \cup K_{1,11} \cup K_{1,22}$ is not a skolem mean graph.

Hence the graph $K_{1,m+2} \cup K_{1,2m+3} \cup K_{1,3m+10}$ is not a skolem mean graph.

Case 9: let $\ell = t = m + 3$.

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+13}$ .let { u }, $\{u_i : 1 \leq i \leq m + 3\}$,{v}, $\{v_j : 1 \leq j \leq 2m + 5\}$ and {w}, $\{w_k : 1 \leq k \leq 3m + 13\}$ be the vertices of $K_{1,m+3}$, $K_{1,2m+5}$ and $K_{1,3m+13}$ respectively.Then $K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+13}$ has 6m + 24 vertices and 6m + 21 edges.

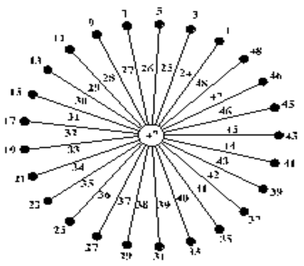
Definef: $V(K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+13}) \rightarrow \{1, 2, 3, \ldots, 6m+24\}$ by f (w) = 6m + 23, $f(w_k) = 2k - 1$, $1 \leq k \leq 3m + 11$ and $f(w_{3m+12}) = 6m + 22$, $f(w_{3m+13}) = 6m + 24$. f (v) = 4, $f(v_j) = 2m + 2j + 10$, $1 \leq j \leq 2m + 5$ andf (u) = 2, $f(u_i) = m + 2i$, $1 \leq i \leq m+3$. The edge label of ww_k is $3m + k + 11, 1 \leq k \leq 3m + 11$, 6m + 23 and 6m + 24. The edge label of vv_j ism + j + 7, $1 \leq j \leq 2m + 5$ and the edge label of uu_i is $\frac{m + 2i + 2}{2}$, $1 \leq i \leq m + 3$.

Hence the induced edge labels 6m + 21 are not receiving distinct edges.

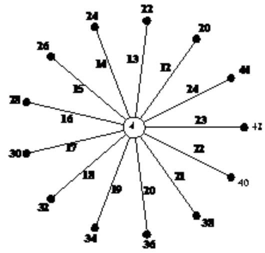
The Skolem mean labeling of $K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+13}$ are illustrated in Fig.2.21, Fig.2.22 and Fig.2.23 respectively.

Consider the graph $G = K_{1,7} \cup K_{1,13} \cup K_{1,25}$ where m = 4.

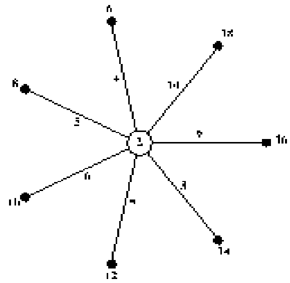
Then | v | = p = 48 and | E | = q = 45.



K1,25



$K_{1,13}$



$K_{1,7}$

Therefore, the edge label of (47,1) is 24 in $K_{1,25}$ and the edge label of (4,44) is 24 in $K_{1,13}$.

Therefore, the two edge labels are same in the graph.

Therefore, the edge labels are not distinct in the graph.

Therefore, the graph $G = K_{1,7} \cup K_{1,13} \cup K_{1,25}$ is not a skolem mean graph.

Hence the graph $K_{1,m+3} \cup K_{1,2m+5} \cup K_{1,3m+13}$ is not a skolem mean graph.

Case 10: let $\ell = t = m + r$ where r = 0, 1, 2, 3, ...

Consider the graph $K_{1,\ell} \cup K_{1,p} \cup K_{1,q} = K_{1,m+r} \cup K_{1,2m+2r-1} \cup K_{1,3m+3r+4}$. Let{u}, $\{u_i : 1 \leq i \leq m + r\}$, {v}, $\{v_j : 1 \leq j \leq 2m + 2r - 1\}$ and { w }, $\{w_k : 1 \leq k \leq 3m + 3r + 4\}$ be the vertices of $K_{1,m+r}$, $K_{1,2m+2r-1}$ and $K_{1,3m+3r+4}$ respectively.Then $K_{1,m+r} \cup K_{1,2m+2r-1} \cup K_{1,3m+3r+4}$ has 6m + 6r + 6 vertices and 6m + 6r + 3 edges.

Definef: $V(K_{1,m+r} \cup K_{1,2m+2r-1} \cup K_{1,3m+3r+4}) \rightarrow \{1, 2, 3, \ldots, 6m + 6r + 6\}$ by f (w)= 6m + 6r + 5, $f(w_k) = 2k-1$, $1 \leq k \leq 3m+3r+2$ and $f(w_{3m+3r+3}) = 6m + 6r + 4$, $f(w_{3m+3r+4}) = 6m + 6r + 6$. f (v) = 4, $f(v_j) = 2m + 2j + 2r + 4$, $1 \leq j \leq 2m+2r-1$ and f(u) = 2, $f(u_i) = m+2i$, $1 \leq i \leq m+r$. The edge label of ww_k is 3m + 3r + k + 2, $1 \leq k \leq 3m + 3r + 2$, 6m + 6r + 5 and 6m

+ 6r + 6. The edge label of vv_j is m + j + r + 4, $1 \leq j \leq 2m + 2r - 1$ and the edge label of uu_i is $\frac{m+2i+2}{2}$, $1 \leq i \leq m + r$. Also, the edge label of ww_1 is 3m + 3r + 3 and the edge label of $vv_{2m+2r-1}$ is 3m + 3r + 3. Therefore, the edge labels are not distinct. Therefore, the induced edge labels 6m + 6r + 3 are not receiving distinct edges. Which is a contradiction. Hence $K_{1,\ell} \cup K_{1,p} \cup K_{1,q}$ is not a skolem mean graph if $| p - q | > 4+\ell$ .Where $\ell = 1, 2, 3, \ldots$ .Hence the theorem.

Conclusion

The communications network addressing: A communication network is composed of nodes, each of which has computing power and can transmit and receive messages over communication links, wireless or cabled. The basic network topologies include fully connected, mesh, star, ring, tree, bus. A single network may consist of several interconnected subnets of different topologies.

Networks are further classified as Local Area Networks (LAN), e.g. inside one building, or Wide Area Networks (WAN), e.g. between buildings. It might beuseful to assign each user terminal a "node label," subject to the constraint that all connecting "edges" (communication links) receive distinct labels. In this way, the numbers of any two communicating terminals automatically specify (by simple subtraction) the link label of the connecting path; and conversely, the path label uniquely specifies the pair of user terminals which it interconnects.Researches may get some information related to graph labeling and its applications in communication field and can get some ideas related to their field of research.

For each kind of application, depending on problem scenario a kind of graph is used for representing the problem. A suitable labeling is applied on that graph in order to solve the problem. Starting from establishing fast and efficient communication.

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