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A Detailed Study on Mean Labeling for Star Graphs and Other Graphs

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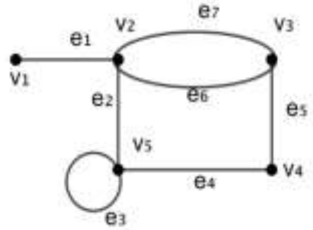
Definition: Graph

A graph G = (V(G), E(G)), consists of two finite sets, V(G), the vertex set of the graph, often denoted by just V, which is non-empty sets of elements called vertices, E(G), the edges set of the graph, often denoted by just E, which is possibly an empty set of element called edges.



A graph G with five vertices and seven edges.

$V(G) = \{V_1, V_2, V_3, V_4, V_5,\}$

$E(G) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$

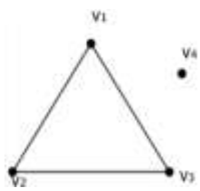
Definition: Empty Graph

An empty graph is graph with no edges.

In the graph empty graph with two vertices.

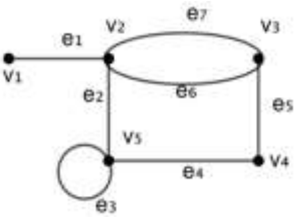
Definition: Isolated

A vertex of G which not end of any edge is called isolated.



Definition: Parallel

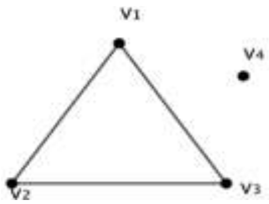
Let G be a graph. If two edges of g have the same end vertices, then these edges are called parallel.



The edges e6 and e7 of the graph of are parallel.

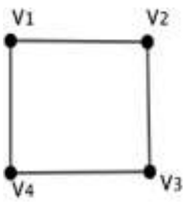
Definition: Adjacent

Two vertices which are joined by an edge are said to be adjacent (or) neighbours. In the graph v_2 and v_3 are adjacent but v_1 and v_4 are not adjacent.



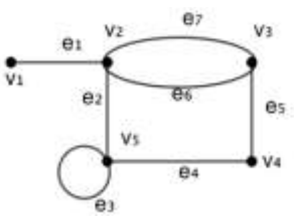
Definition: Simple Graph

A graph is called simple if it has no loops and no parallel edges.



Definition: Multigraph

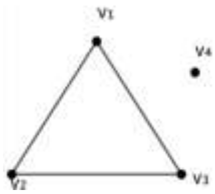
A graph which is not simple is called a multigraph.



In the graph, G is a multigraph.

Definition: Neighbourhood Set

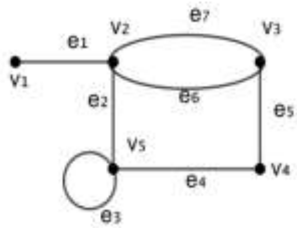
The set of all neighbours of a fixed v of G is called the neighbourhood set of v and is denoted by N(v).



In the graph of the neighbourhood set $N(v_1)$ of v_1 is $\{v_2, v_3\}$.

Definition: Loop

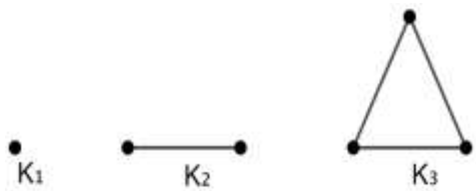
It is possible to have a vertex u joined to by an edge, such an edge is called as a loop.



In the graph the vertex v_5 has the loop.

Definition: Complete Graph

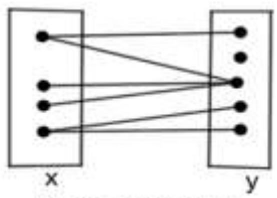
A complete graph is a simple graph in which each pair of distinct vertices is joined by an edge. It is denoted by K_n.



In the complete graph with one, two, and three vertices.

Definition: Bipartite Graph

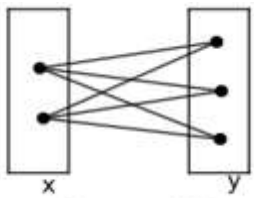
A graph G is trivial if its vertex set is singleton and it contains no edges. A graph is bipartite if its vertex set can be partitioned into two nonempty subset X and Y such that each edge of G has one end in X and the other in Y. the pair (X, Y) is called a bipartition of the bipartite graph. The bipartite graph G with bipartition (X, Y) is denoted by G(X ,Y).



A bipartite graph

Definition: Complete Bipartite Graph

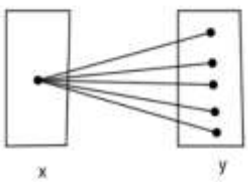
A simple bipartite $G(X,Y)$ is complete if each vertex of X is adjacent to all the other vertices of Y. If $G(X,Y)$ is complete with │X│= p and │Y│= q, then $G(X,Y)$ is denoted by $K_{p,q}$.



The graph $K_{2,3}$

Definition: Star Graph

A complete bipartite graph of the form $K_{1,\ q}$ is called a star.

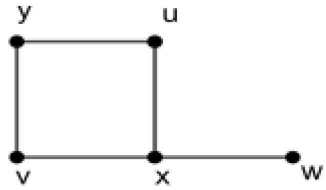


The graph $K_{2,3}$

Definition: Vertex Independent Sets

A subset S of the vertex set V of a graph G is called independent if no two vertices of S are adjacent in G. S $\subseteq$ V is a maximum independent set of G if G has no independent set S' with │S'│>│S│. A maximum independent set that is not is a proper subset of another independent set of G.

For example, in the graph of figure $\{u,v,w\}$ is a maximum independent set and $\{x,y\}$ is maximal of that is not maximum.

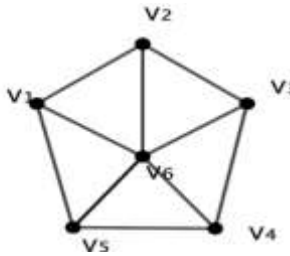


{u, v, w} → maximum independent set.
{x, y} → maximum independent set.

Definition: Covering

A subset k of V is called a covering of G if every edge of G is incident with at least one vertex of k. A covering k is minimum if there is no covering k' of G such that │k'│<│k│it is minimal if there is no covering k_1 of G such that k_1 is a proper subset of k.

Example:



$\{v_6\}$ is the minimum covering of the figure.

In the graph w_5 of figure $\{v_1, v_2, v_3, v_4, v_5\}$ is a covering of w_5 and$\{v_1, v_3, v_4, v_6,\}$ is a minimal covering. Also the set $\{x, y\}$ is a minimum covering of the graph of figure.

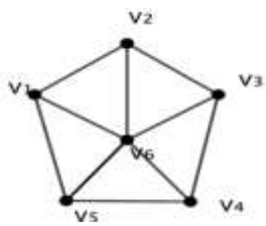
Definition:Edge Independent Set

1. A subset M of the edge set E of a loopless graph G is called independent if no two edges of M are adjacent in G.
2. A matching in G is a set of independent edge.
3. An edge covering of G is a subset L of E such that every vertex of G is incident to some edge of L. Hence an edge covering of G exists if $\delta > 0$.
4. A matching M of G is maximum if G has no matching M' with │M'│>│M│. M is maximum strictly containing M. ά(G) is the cardinality of a maximum matching and β'(G) is the size of a minimum edge covering of G.
5. A set of vertices of G is said to be saturated by a matching M of G or M-saturated if every vertex of S is incident to some edge of M.A vertex v of

G is M-saturated if {v} is M-saturated. V is M-unsaturated if it is not M- saturated.

Definition: Augmenting Path

An M-augmenting path in G is a path in which the edges alternate between E/M and M and its end vertices are M-saturated. AnM-alternating path in G is a path whose edges alternate between E/M and M.



Example:

In the graph G of the above figure, M_1 = $\{v_1v_2, v_3v_4, v_5v_6\}$ and, $M_2 = \{v_1v_2, v_3v_6, v_4v_5\}$ and, $M_3 = \{v_3v_4, v_5v_6\}$ are matching of G. The path $v_2v_3v_4v_6v_5v_1$ is an M_3- augmenting path in G.

Definition: Matching

A matching of a graph G is a set of independent edges of G.

If $e = uv$ is an edges of a matching M of G, the end vertices u and v of e are said to be matched by M.

If M_1 and M_2 are matching of G, the edge subgraph defined by $M_1 \Delta M_2$, the symmetric difference of M_1 and M_2 is a subgraph H of G whose components are paths or even cycles of G in which the edges alternative between M_1 and M_2

A matching of a graph G is a set of independent edges of G. If $e = uv$ is an edge of a matching M of G, the end vertices u and v are said to be matched by M.

Definition: Perfect Matching

A matching M is called a perfect matching if every point of G is M-saturated M is called a maximum matching if there is no matching M' in G such $|M'| \leq |M|$

Example

Consider the graph G_1 gives in figure $M_1 = \{v_1v_2, v_6v_3, v_5v_4\}$ is a perfect matching in G_1. Also $M_2 = \{v_1v_3, v_6v_5\}$ is matching in G_1. However M_2 is not a perfect matching since the vertices v_2 and v_4 are not M_2 - saturated.

For the graph G_2 given infigure $M = \{v_1v_2, v_8v_4\}$ is a maximum matching but it is not a perfect matching.

For the G_1 given in a figure $P_1 = \{v_6, v_5, v_4, v_3\}$ is an M_1- alternating path also $P_2 = \{v_2, v_1, v_3, v_6, v_5, v_4\}$ is an M_2 - alternating path.

Definition: Perfect Matching

A factor a graph G is spanning subgraph of G. A k- factor of G is a factor of G this is k- regular. Thus 1- factor of G is a matching that Saturates all the vertices of G and 1- factor of G is perfect matching if G.

For example, in the wheel (fig 1) $M = \{v_1v_2, v_4v_6\}$ is a maximal matching; $\{v_1v_5, v_2v_3, v_4v_6\}$ is a maximum matching and a minimum edge covering the vertices v_1, v_2, v_4 and v_6 are M-saturated whereas v_3 and v_5 are M-unsaturated.

Non Existence of Relaxed Mean Labelingfor Subdivision of Star Graphs

In this chapter we prove that the subdivision of star G = S($K_{1,n}$) for n >5 is not a relaxed mean graph.

Theorem

$S(K_{1,n})$ for n > 5 is not a relaxed mean graph.

Proof

Suppose $S(K_{1,6})$ is a relaxed mean graph with labeling f and induced edge labeling f*.

Let $V(S(K_{1,6}))=\{u,v_i,w_i: 1\leq i\leq 6\}$ and $E(S(K_{1,6})) =\{uv_i, v_iw_i:1\leq i\leq 6\}$. Then $S(K_{1,6})$ has p = 13 vertices and q = 12 edges.

The vertex labels are from 0, 1, 2, …, 13. As there are 13 vertices, one of {0, 1, 2, …, 13} is not a label of any vertex. The induced edge labels are from {1, 2, …, 12} and should be distinct. As there are 12 edges, the first 12 positive integers are labels of the edges. Only the pair 0, 1 or 0, 2 can give the edge label 1 and hence 0 must be a label of a vertex. Therefore one of {1, 2, …, 13} is not a label of any vertex. Another observation is that if a vertex has label which is an odd number then the two vertices adjacent to it cannot have the labels respectively 2i and 2i + 1 for any i. Similarly, if a vertex has label which is an even number then the two vertices adjacent to it cannot have the labels respectively 2i - 1 and 2i for any i.

Let us assume first that f(u) = 0.

Case 1 f(u) = 0.

Since 0 is an even number, the possible labels for the vertices $v_i, 1\leq i \leq 6$ are 1 or 2, 3 or 4, 5 or 6, 7 or 8, 9 or 10, 11 or 12 and 13. Hence there are seven possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \leq i \leq 6\right)$ are 1, 2, 3, 4, 5, 6 and 7.

Case 1 (a): f(u) = 0 and suppose 13 is not a label of any vertex.

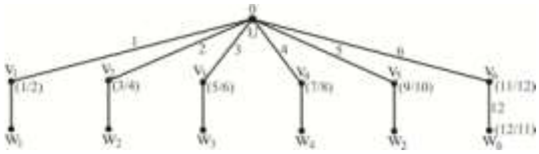
Hence, **(i)** the labels for the vertices v_i, $1 \leq i \leq 6$ are (respectively) 1 or 2, 3 or 4, 5 or 6, 7 or 8, 9 or 10, 11 or 12.

(**ii)** The possibility to get the edge label 12,

a) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(v_6) = 11$ and $f(w_6) = 12$.

The induced edge labels of $uv_i, 1 \leq i \leq 6$ are 1, 2, 3, 4, 5, 6 and $f^*(v_6w_6)$ =12. Hence we have the following figure

.



The labels of the remaining edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 are 7, 8, 9, 10 and 11. We have already used the labels 0, 11 and 12 for the vertices and got induced edge labels 1, 2, 3, 4, 5, 6 and 12. The remaining edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 should have the labels 7, 8, 9, 10 and 11. Now the next maximum possible label for any edge from $\{v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5\}$ is 10 (if the adjacent vertices have the labels 9 and 10). Therefore, it is not possible to have the label 11 for any edge.
Hence 13 is necessarily a label of a vertex.

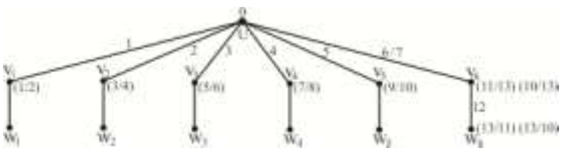
Case 1 (b): f(u) = 0 and suppose 12 is not a label of any vertex.

Hence, **(i)** the labels for the vertices v_i, $1 \leq i \leq 6$ are from 1 or 2,3 or 4, 5 or 6, 7or 8, 9 or 10, 11 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.



In either of the cases we would have used '10 and 13' or '11 and 13'. The remaining two largest numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively and the maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 0 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 1 or 2,3 or 4, 5 or 6, 7 or 8, 9 or 10, 12 and 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(iii) The possibility to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The remaining two largest numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But, we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f(u) = 0 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 1 or 2,3 or 4, 5 or 6, 7 or 8, 9, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The remaining two largest numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 0 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 1 or 2, 3 or 4, 5 or 6, 7 or 8, 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 8$ and $f(w_5) = 13$ or $f(w_5) = 8$ and $f(v_5) = 13$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

d) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

e) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The remaining two largest numbers are 8 and 7 and the maximum possible edge label hereafter is 8, but we have not got so far the edge label 9.

Therefore, 9 is also necessarily a vertex label.

Case 1 (f): f(u) = 0 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 1 or 2, 3 or 4, 5 or 6, 7, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to have the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In ii) and iii), suppose we have used 10, 11, 12 and 13, Then the remaining two largest numbers are 9 and 7 and the maximum possible edge label hereafter is 8, but we have not got so far the edge label 10and 9.

Therefore, 8 is also necessarily a vertex label.

Case 1 (g): f(u) = 0 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 1 or 2, 3 or 4, 5 or 6, 8, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to have the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

e) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 13$ and $f(w_3) = 4$ or $f(w_3) = 13$ and $f(v_3) = 4$.

c) Either $f(v_3) = 12$ and $f(w_3) = 6$ or $f(w_3) = 12$ and $f(v_3) = 6$.

d) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

e) Either $f(v_3) = 11$ and $f(w_3) = 6$ or $f(w_3) = 11$ and $f(v_3) = 6$.

f) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

g) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The remaining two largest numbers are 6 and 5 and the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8.

Therefore, 7 is also necessarily a vertex label.

Case 1 (h): f(u) = 0 and suppose 6 is not a label of any vertex.

Let us fix $f(v_6) = 13$ and $f(w_6) = 11$, to get the edge label 12.

Now, the remaining vertex labels are $\{1, 2, 3, 4, 5, 7, \ldots, 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 1 or 2, 3 or 4, 5, 7 or 8, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iii) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(iv) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The remaining two largest numbers are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex label 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, …, 13} is to be labeled necessarily.

This is a contradiction. Hence $S(K_{1,6})$ is not a relaxed mean graph when f(u) = 0.

Next, Let us assume that f(u) = 1.

Case 1 f(u) = 1.

Since 1 is an odd number, the possible labels for the vertices v_i, $1 \le i \le 6$ are 0, 2 or 3, 4 or 5, 6 or 7, 8 or 9, 10 or 11 and 12 or 13. Hence there are seven possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \le i \le 6\right)$ are 1, 2, 3, 4, 5, 6 and 7.

Case 1 (a): f(u) = 1 and suppose 13 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are 0, 2 or 3, 4 or 5, 6 or 7, 8 or 9, 10 or 11 and 12.

(ii) The possibility to get the edge label 12,

a) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

The induced edge labels of $v_i, 1 \le i \le 6$ are 1, 2, 3, 4, 5, 6, 7 and $f^*(v_6w_6) = 12$.

The labels of the remaining edges (v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5) are 8, 9, 10 and 11 and one among 2 to 7. Now the maximum possible label for any edge from {v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5} is 10 (if the adjacent vertices have the labels 9 and 10). Therefore, it is not possible to have the label 11 for any edge.

Hence 13 is necessarily a label of a vertex.

Case 1 (b): f(u) = 1 and suppose 12 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3, 4 or 5, 6 or 7, 8 or 9, 10 or 11 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

In either of the cases we would have used '10 and 13' or '11 and 13'. The largest two remaining numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively. The maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 1 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3,4 or 5, 6 or 7, 8 or 9, 10 and 12 or 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(iii) The possibility to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f(u) = 1 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3, 4 or 5, 6 or 7, 8 or 9, 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(v_5) = 9$ and $f(w_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 1 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3, 4 or 5, 6 or 7, 8, 10 or 11 and 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

d) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

e) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The remaining two largest numbers are 8 and 7, the maximum possible edge label hereafter is 8, but we have not got so far the edge label 9.

Therefore, 9 is also necessarily a vertex label.

Case (f): f(u) = 1 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3, 4 or 5, 6 or 7, 9, 10 or 11 and 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In two cases **(ii & iii)** we would have used 10, 11, 12 and 13. The largest two remaining numbers are 9 and 7 according as we use the numbers. The maximum possible edge label hereafter is 8, but we have not got so far the edge label 10 and 9.

Therefore, 8 is also necessarily a vertex label.

Case 1 (g): f(u) = 1 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3, 4 or 5, 6, 8 or 9, 10 or 11 and 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

e) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 13$ and $f(w_3) = 4$ or $f(w_3) = 13$ and $f(v_3) = 4$.

c) Either $f(v_3) = 12$ and $f(w_3) = 6$ or $f(w_3) = 12$ and $f(v_3) = 6$.

d) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

e) Either $f(v_3) = 11$ and $f(w_3) = 6$ or $f(w_3) = 11$ and $f(v_3) = 6$.

f) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

g) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The remaining two largest numbers are 6 and 5, the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8.

Therefore, 7 is also necessarily a vertex label.

Case 1 (h): f (u) = 1 and suppose 6 is not a label of any vertex.

Let us fix $f(v_6) = 13$ and $f(w_6) = 11$, to get the edge label 12.

Now, the remaining vertex labels are $\{0, 2, 3, 4, 5, 7, \dots, 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 2 or 3, 4 or 5, 7, 8 or 9, 10 or 11 and 12 or 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iii) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(iv) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The remaining largest numbers possible are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex label 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, … , 13} is to be labeled necessarily.

This is a contradiction. Hence $S(K_{1,6})$ is not a relaxed mean graph when f(u) = 1.

Next, Let us assume that f(u) = 2.

Case 1 f(u) = 2.

Since 2 is an even number, the possible labels for the vertices $v_i, 1 \le i \le 6$ are 0, 1, 3 or 4, 5 or 6, 7 or 8, 9 or 10, 11 or 12 and 13.There are eight possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \le i \le 6\right)$ are 1, 2, 3, 4, 5, 6, 7 and 8.

Case 1 (a): f(u) = 2 and suppose 13 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are (respectively) 0, 1, 3 or 4, 5 or 6, 7 or 8, 9 or 10, 11 or 12.

(ii) The possibility to get the edge label 12,

(a) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

The induced edge labels of $uv_i, 1 \le i \le 6$ are 1, 2, 3, 4, 5, 6, 7 and $f^*(v_6w_6) = 12$.

The remaining edge labels (v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5) are 8, 9, 10, 11 and one among 2 to 7. We have already used the labels 2, 11 and 12 for the vertices and induced edge labels are 1, 2, 3, 4, 5, 6 and 12. The remaining edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 should have the labels one among 2 to 7, 8, 9, 10 and 11. Now the maximum possible label for any edge from {v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5} is 10 (if the adjacent vertices have the labels 9 and10). Therefore it is not possible to have the label 11 for any edge. Hence 13 is necessarily a label of a vertex.

Case 1 (b): f(u) = 2 and suppose 12 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or 4, 5 or 6, 7 or 8, 9 or 10, 11 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

In either of the cases we would have used '10 and 13' or '11 and 13'. The largest two remaining numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively. And the maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 2 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or 4, 5 or 6, 7 or 8, 9 or 10, 11 or 12 and 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(ii) The possibility to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f(u) = 2 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or 4, 5 or 6, 7 or 8, 9, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 2 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or 4, 5 or 6, 7 or 8, 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

d) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

e) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The remaining two largest numbers are 8 and 7, the maximum possible edge label hereafter is 8, but we have not got so far the label 9 for any edge.

Therefore, 9 is also necessarily a vertex label.

Case 1 (f): f(u) = 2 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or 4, 5 or 6, 7, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In two cases (ii & iii) we would have used 10, 11, 12 and 13. The largest two remaining numbers are 9 and 7 according as we use the numbers. The maximum possible edge label hereafter is 8, but we have not got edge label 10 and 9.

Therefore, 8 is also necessarily a vertex label.

Case 1 (g): f(u) = 2 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or 4, 5 or 6, 8, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

e) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 13$ and $f(w_3) = 4$ or $f(w_3) = 13$ and $f(v_3) = 4$.

c) Either $f(v_3) = 12$ and $f(w_3) = 6$ or $f(w_3) = 12$ and $f(v_3) = 6$.

d) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

e) Either $f(v_3) = 11$ and $f(w_3) = 6$ or $f(w_3) = 11$ and $f(v_3) = 6$.

f) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

g) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The remaining two largest numbers are 6 and 5, the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8.

Therefore, 7 is also necessarily a vertex label.

Case 1(h): f(u) = 2 and suppose 6 is not a label of any vertex.

Let us fix $f(v_6) = 13$ and $f(w_6) = 11$, to get the edge label 12.

Now, the remaining vertex labels are $\{0, 1, 3, 4, 5, 7, ..., 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1, 3 or4, 5, 7 or 8, 9 or 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iii) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(iv) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The remaining two largest numbers are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex label 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, … , 13} is to be labeled necessarily.

This is a contradiction. Hence $S(K_{1, 6})$ is not a relaxed mean graph when f(u) = 2.

Next, Let us assume that f(u) = 3.

Case 1 f(u) = 3.

Since 3 is an odd number, the possible labels for the vertices $v_i, 1 \le i \le 6$ are 0 or 1, 2, 4 or 5, 6 or 7, 8 or 9, 10 or 11, 12 or 13. Hence there are seven possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \le i \le 6\right)$ are 2, 3, 4, 5, 6, 7 and 8.

Case 1 (a): f(u) = 3 and suppose 13 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are 0 or 1, 2,3, 4 or 5, 6 or 7, 8 or 9, 10 or 11, 12.

(ii) The possibility to get the edge label 12,

a) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

The induced edge labels of $uv_i, 1 \le i \le 6$ are 2, 3, 4, 5, 6, 7, 8 and $f^*(v_6w_6) = 12$.

The labels of the remaining edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 are 8, 9, 10, 11, 1 and one among 2 to 7. We have already used the labels 2, 11 and 12 for the vertices and the induced edge labels are 1, 2, 3, 4, 5, 6 and 12.

The remaining edge labels (v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5) are 1, one among 2 to 8, 9, 10 and 11. Now the maximum possible label for any edge from {v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5} is 10 (if the adjacent vertices have the labels 9 and 10). Therefore, it is not possible to have the label 11 for any edge.

Hence 13 is necessarily a label of a vertex.

Case 1 (b): f(u) = 3 and suppose 12 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 8 or 9, 10 or 11, 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

In either of the cases we would have used '10 and 13' or '11 and 13'. The largest two remaining numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively and the maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 3 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 8 or 9, 10, 12 or 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(iii) The possibility to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f(u) = 3 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 8 or 9, 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 3 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 8, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

d) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

e) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The next possible two largest numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so the edge label 9.

Therefore, 9 is also necessarily a vertex label.

Case 1 (f): f(u) = 3 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 9, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In two cases (ii & iii) we would have used 10, 11, 12 and 13. The largest two remaining numbers are 9 and 7 according as we use the numbers. The maximum possible edge label hereafter is 8, but we have not got so for the edge label 10 and 9.

Therefore, 8 is also necessarily a vertex label

Case 1 (g): f(u) = 3 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 8 or 9, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

e) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 13$ and $f(w_3) = 4$ or $f(w_3) = 13$ and $f(v_3) = 4$.

c) Either $f(v_3) = 12$ and $f(w_3) = 6$ or $f(w_3) = 12$ and $f(v_3) = 6$.

d) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

e) Either $f(v_3) = 11$ and $f(w_3) = 6$ or $f(w_3) = 11$ and $f(v_3) = 6$.

f) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

g) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 6 and 5, the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8.

Therefore, 7 is also necessarily a vertex label.

Case 1 (h): f(u) = 3 and suppose 6 is not a label of any vertex.

Let us fix $f(v_6) = 13$ and $f(w_6) = 11$, to gives the edge label 12, the remaining vertex labels are $\{0, 1, 2, 4, 5, 7, \ldots, 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2, 4 or 5, 6 or 7, 8 or 9, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iii) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(iv) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex label 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, … , 13} is to be labeled necessarily.

This is a contradiction. Hence $S(K_{1,6})$ is not a relaxed mean graph when f(u) = 3.

Next, Let us assume that f(u) = 4.

Case 1 f(u) = 4.

Since 4 is an even number, the possible labels for the vertices $v_i, 1 \le i \le 6$ are 0, 1 or 2, 3, 5 or 6, 7 or 8, 9 or 10, 11 or 12 and 13. Hence there are eight possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \le i \le 6\right)$ are 2, 3, 4, 5, 6, 7, 8 and 9.

Case 1 (a): f(u) = 4 and suppose 13 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are 0, 1 or 2, 3, 5 or 6, 7 or 8, 9 or 10, 11 or 12 and 13.

(ii) The possibility to get the edge label 12,

(a) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

The induced edge labels of $uv_i, 1 \le i \le 6$ are 2, 3, 4, 5, 6, 7, 8, 9 and $f^*(v_6w_6) = 12$.

The remaining edge labels (v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5) are 1, two among 2 to 9, 10 and 11. We have already used the labels 4, 11 and 12 for the vertices and the induced edge labels are 1, 2, 3, 4, 5, 6 and 12. The remaining edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 should have the labels, 1, two among 2 to 9, 10 and 11. Now the maximum possible label for any edge from $\{v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5\}$ is 10 (if the adjacent vertices have the labels 9 and 10). Therefore, it is not possible to have the label 11 for any edge.

Hence 13 is necessarily a label of a vertex.

Case 1 (b): f(u) = 4 and suppose 12 is not a label of any vertex.Hence,

(i) the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 7 or 8, 9 or 10, 11 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

In either of the cases we would have used '10 and 13' or '11 and 13'. The largest two remaining numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively and the maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 4 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices v_i, $1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 7 or 8, 9 or 10, 12 and 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(iii) The possibility to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The remaining two largest numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f(u) = 4 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 7 or 8, 9, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The remaining two largest numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 4 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 7 or 8, 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

d) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

e) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The remaining two largest numbers are 8 and 7, the maximum possible edge label hereafter is 8, but we have not got so far the edge label 9.

Therefore, 9 is also necessarily a vertex label.

Case 1 (f): f(u) = 4 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 7, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In two cases (ii & iii) we would have used 10, 11, 12 and 13. The largest two remaining numbers are 9 and 7 according as we use the numbers. The maximum possible edge label hereafter is 8, but we have not got so for edge the label 10 and 9.

Therefore, 8 is also necessarily a vertex label.

Case 1 (g): f(u) = 4 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 8, 9 or 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

e) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 12$ and $f(w_3) = 6$ or $f(w_3) = 12$ and $f(v_3) = 6$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 11$ and $f(w_3) = 6$ or $f(w_3) = 11$ and $f(v_3) = 6$.

e) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

f) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 6 and 5, the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8.

Therefore, 7 is also necessarily a vertex label.

Case 1 (h): f(u) = 4 and suppose 6 is not a label of any vertex.

Suppose we fix $f(v_6) = 13$ and $f(w_6) = 11$, to get the edge label 12.

Now, the remaining vertex labels are $\{0, 1, 2, 3, 5, ..., 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5, 7 or 8, 9 or 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iii) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(iv) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex label 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, … , 13} is to be labeled necessarily.

This is a contradiction. Hence $S(K_{1,6})$ is not a relaxed mean graph when f(u) = 4.

Next, Let us assume that f(u) = 5.

Case 1 f(u) = 5.

Since 5 is an odd number, the possible labels for the vertices $v_i, 1 \le i \le 6$ are 0 or 1, 2 or 3, 4, 6 or 7, 8 or 9, 10 or 11, 12 or 13. Hence there are seven possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \le i \le 6\right)$ are 3, 4, 5, 6, 7, 8 and 9.

Case 1(a): f(u) = 5 and suppose 13 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are (respectively) 0 or 1, 2 or 3, 4, 6 or 7, 8 or 9, 10 or 11, 12.

(ii) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

The induced edge labels of $uv_i, 1 \le i \le 6$ are 3, 4, 5, 6, 7, 8, 9 and $f^*(v_6w_6)$ =12.

The remaining edge labels (v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5) are1, 2, one among 3 to 9, 10 and 11. We have already used the labels 5, 11 and 12 for the vertices and got induced edge labels 3, 4, 5, 6, 7, 8, 9 and 12.

There maiming edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 should have the labels, 1, 2, one among 3 to 9, 10 and 11. Now, the maximum possible label for any edge from {v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5} is 10(if the adjacent vertices have the labels 9 and 10). Therefore, it is not possible to have the label 11 for any edge.

Hence 13 is necessarily a label of a vertex.

Case 1 (b): f(u) = 5 and suppose 12 is not a label of any vertex. Hence,

(i) the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1,2 or 3, 4, 6 or 7, 8 or 9, 10 or 11, 13 respectively.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

In either of the cases we would have used '10 and 13' or '11 and 13'. The largest two remaining numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively and the maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 5 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2 or 3, 4, 6 or 7, 8 or 9, 10, 12 or 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(iii) The possibility to get the edge label 11,

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f(u) = 5 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2 or 3, 4, 6 or 7, 8 or 9, 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(v_5) = 9$ and $f(w_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 5 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2 or 3, 4, 6 or 7, 8, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

d) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

e) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The next possible two largest numbers are 8 and 7, the maximum possible edge label hereafter is 8, but we have not got so far the edge label 9.

Therefore, 9 is also necessarily a vertex label.

Case 1 (f): f(u) = 5 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \leq i \leq 6$ are from 0 or 1, 2 or 3, 4, 6 or 7, 9, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In two cases (ii & iii) we would have used 10, 11, 12 and 13. The largest two remaining numbers are 9 and 7 according as we use the numbers. The maximum possible edge label hereafter is 8, but we have not got so far the edge label 10 and 9.

Therefore, 8 is also necessarily a vertex label.

Case 1 (g): f(u) = 5 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \leq i \leq 6$ are from 0 or 1, 2 or 3, 4, 6, 8 or 9, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 6$ or $f(w_4) = 13$ and $f(v_4) = 6$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

e) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 12$ and $f(w_3) = 6$ or $f(w_3) = 12$ and $f(v_3) = 6$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 11$ and $f(w_3) = 6$ or $f(w_3) = 11$ and $f(v_3) = 6$.

e) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

f) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 6 and 5, the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8. Therefore, 7 is also necessarily a vertex label.

Case 1 (h): f(u) = 5 and suppose 6 is not a label of any vertex.

Suppose we fix $f(v_6) = 13$ and $f(w_6) = 11$, to get the edge label 12.

Now, the remaining vertex labels are $\{0, 1, 2, 3, 4, 6, ..., 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0 or 1, 2 or 3, 4, 7, 8 or 9, 10 or 11, 12 or 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iii) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(iv) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex label 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, … , 13} is to be labeled necessarily.

This is a contradiction. Hence $S(K_{1, 6})$ is not a relaxed mean graph when f(u) = 5.

Next, Let us assume that f(u) = 6.

Case 1 f(u) = 6.

Since 6 is an even number, the possible labels for the vertices $v_i, 1 \le i \le 6$ are 0, 1 or 2, 3 or 4, 5, 7 or 8, 9 or 10, 11 or 12 and 13. Hence there are eight possibilities for six labels. The corresponding edge labels $\left(\text{of } uv_i, 1 \le i \le 6\right)$ are 3, 4, 5, 6, 7, 8, 9 and 10.

Case 1 (a): f (u) = 6 and suppose 13 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are (respectively) 0, 1 or 2, 3 or 4, 5, 7 or 8, 9 or 10, 11 or 12 and 13.

(ii) The possibility to get the edge label 12,

(a) $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

The induced edge labels of $uv_i, 1 \le i \le 6$ are 3, 4, 5, 6, 7, 8, 9, 10 and $f^*(v_6w_6) = 12$.

The remaining edge labels (v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5) are 1, 2, two among 3 to 10 and 11. We have already used the labels 6, 11 and 12 for the vertices and got induced edge labels 3, 4, 5, 6, 7, 8 and 12. The remaining edges v_1w_1, v_2w_2, v_3w_3, v_4w_4 and v_5w_5 should have the labels, 1, 2, two among 3 to 10 and 11. Now, the maximum possible label for any edge from {v_1w_1, v_2w_2, v_3w_3, v_4w_4, v_5w_5} is 10 (if the adjacent vertices have the labels 9 and 10). Therefore, it is not possible to have the label 11 for any edge.

Hence 13 is necessarily a label of a vertex.

Case 1 (b): f(u) = 6 and suppose 12 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3 or 4, 5, 7 or 8, 9 or 10, 11 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

In either of the cases we would have used '10 and 13' or '11 and 13'. The largest two remaining numbers are 9 and 11 or 9 and 10 according as we use 10 or 11 respectively. And the maximum possible edge label hereafter is 10. But we have not got so far the label 11 for any edge.

Hence 12 also is necessarily a label of a vertex.

Case 1 (c): f(u) = 6 and suppose 11 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3 or 4, 5, 7 or 8, 9 or 10, 12 and 13.

(ii) The possibility to get the edge label 12,

a) Either $f(v_6) = 10$ and $f(w_6) = 13$ or $f(w_6) = 10$ and $f(v_6) = 13$.

(iii) The possibility to get the edge label 11,

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 9 and 10 for any edge.

Hence 11 also is necessarily a label of a vertex.

Case 1 (d): f (u) = 6 and suppose 10 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3 or 4, 5, 7 or 8, 9, 11 or 12 and 13

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 11$ and $f(w_6) = 13$ or $f(w_6) = 11$ and $f(v_6) = 13$.

b) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

b) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

The largest two remaining numbers are 8 and 7 and the maximum possible edge label hereafter is 8. But we have not got so far the label 10 and 9 for any edge.

Hence 10 also is necessarily a label of a vertex.

Case 1 (e): f(u) = 6 and suppose 9 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3 or 4, 5, 7 or 8, 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 13$ and $f(w_4) = 7$ or $f(w_4) = 13$ and $f(v_4) = 7$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

d) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

The next possible two largest numbers are 8 and 7, the maximum possible edge label hereafter is 8, but we have not got so far the edge label 9.

Therefore, 9 is also necessarily a vertex label.

Case 1 (f): f (u) = 6 and suppose 8 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3 or 4, 5, 7, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 12$ and $f(w_6) = 11$ or $f(w_6) = 12$ and $f(v_6) = 11$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 10$ and $f(w_5) = 12$ or $f(w_5) = 10$ and $f(v_5) = 12$.

b) Either $f(v_5) = 10$ and $f(w_5) = 11$ or $f(w_5) = 10$ and $f(v_5) = 11$.

c) Either $f(v_5) = 9$ and $f(w_5) = 13$ or $f(w_5) = 9$ and $f(v_5) = 13$.

d) Either $f(v_5) = 9$ and $f(w_5) = 12$ or $f(w_5) = 9$ and $f(v_5) = 12$.

In two cases (ii & iii) we would have used 10, 11, 12 and 13. The largest two remaining numbers are 9 and 7 according as we use the numbers. The maximum possible edge label hereafter is 8, but we have not got so far the edge label 10 and 9.

Therefore, 8 is also necessarily a vertex label.

Case 1 (g): f (u) = 6 and suppose 7 is not a label of any vertex.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5 or 6, 8, 9 or 10, 11 or 12 and 13.

(ii) The possibilities to get the edge label 12,

a) Either $f(v_6) = 13$ and $f(w_6) = 11$ or $f(w_6) = 13$ and $f(v_6) = 11$.

b) Either $f(v_6) = 13$ and $f(w_6) = 10$ or $f(w_6) = 13$ and $f(v_6) = 10$.

c) Either $f(v_6) = 11$ and $f(w_6) = 12$ or $f(w_6) = 11$ and $f(v_6) = 12$.

(iii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 13$ and $f(w_5) = 9$ or $f(w_5) = 13$ and $f(v_5) = 9$.

b) Either $f(v_5) = 13$ and $f(w_5) = 8$ or $f(w_5) = 13$ and $f(v_5) = 8$.

c) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

d) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

e) Either $f(v_5) = 11$ and $f(w_5) = 10$ or $f(w_5) = 11$ and $f(v_5) = 10$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 12$ and $f(w_4) = 8$ or $f(w_4) = 12$ and $f(v_4) = 8$.

b) Either $f(v_4) = 11$ and $f(w_4) = 9$ or $f(w_4) = 11$ and $f(v_4) = 9$.

c) Either $f(v_4) = 11$ and $f(w_4) = 8$ or $f(w_4) = 11$ and $f(v_4) = 8$.

d) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 13$ and $f(w_3) = 5$ or $f(w_3) = 13$ and $f(v_3) = 5$.

b) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

c) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 6 and 5, the maximum possible edge label hereafter is 6, but we have not got so far the edge label 7 and 8.

Therefore, 7 is also necessarily a vertex label.

Case 1 (h): f(u) = 6 and suppose 6 is not a label of any vertex.

Suppose we fix $f(v_6) = 13$ and $f(w_6) = 11$, to gives the edge label 12.

Now, the remaining vertex labels are $\{0, 1, 2, 3, 4, 5, 7, ..., 10, 12\}$.

Hence, **(i)** the labels for the vertices $v_i, 1 \le i \le 6$ are from 0, 1 or 2, 3, 5, 7 or 8, 9 or 10, 11 or 12, and 13.

(ii) The possibilities to get the edge label 11,

a) Either $f(v_5) = 12$ and $f(w_5) = 10$ or $f(w_5) = 12$ and $f(v_5) = 10$.

b) Either $f(v_5) = 12$ and $f(w_5) = 9$ or $f(w_5) = 12$ and $f(v_5) = 9$.

(iv) The possibilities to get the edge label 10,

a) Either $f(v_4) = 10$ and $f(w_4) = 9$ or $f(w_4) = 10$ and $f(v_4) = 9$.

b) Either $f(v_4) = 12$ and $f(w_4) = 8$ Or $f(w_4) = 12$ and $f(v_4) = 8$.

c) Either $f(v_4) = 12$ and $f(w_4) = 7$ or $f(w_4) = 12$ and $f(v_4) = 7$.

(v) The possibilities to get the edge label 9,

a) Either $f(v_3) = 10$ and $f(w_3) = 8$ or $f(w_3) = 10$ and $f(v_3) = 8$.

b) Either $f(v_3) = 10$ and $f(w_3) = 7$ or $f(w_3) = 10$ and $f(v_3) = 7$.

c) Either $f(v_3) = 12$ and $f(w_3) = 5$ or $f(w_3) = 12$ and $f(v_3) = 5$.

d) Either $f(v_3) = 9$ and $f(w_3) = 8$ or $f(w_3) = 9$ and $f(v_3) = 8$.

The next possible two largest numbers are 5 and 4, the maximum possible edge label hereafter is 5, but we have not got so far the edge label 8, 7 and 6.

Therefore, 6 is also necessarily a vertex label.

Similarly, we can prove that the vertex labels 5, 4, 3, 2, 1 is also necessary. Therefore, each element of the vertex set {0, 1, 2, 3, …, 13} is to be labeled necessarily.

This is a contradiction. Therefore, $S(K_{1,6})$ is not a relaxed mean graph when f(u) = 6. Therefore, $S(K_{1,6})$ is not a relaxed mean graph.

Similarly, we can prove that $S(K_{1,7})$ is not a relaxed mean graph.

Hence $S(K_{1,n})$ for n > 5 is not a relaxed mean graph.

Hence the theorem.

References

1. Bermond J.C, **Graceful graphs, radio antennae and French windmills**, Graph theory and Combinatorics, Pitman, London, (1979), 13 – 37.
2. Balaji V, Ramesh D.S.T and Subramanian A, **Skolem Mean Labeling,** Bulletin of Pure and Applied Sciences, vol. 26E No. 2, 2007, 245 – 248.
3. Balaji V, Ramesh D.S.T and Subramanian A, **Some Results on Skolem Mean Graphs,** Bulletin of Pure and Applied Sciences, vol. 27E No. 1, 2008, 67 – 74.
4. Balaji V, Ramesh D.S.T and Subramanian A, **Relaxed Skolem Mean Labeling,** Advances and Applications in Discrete Mathematics, vol. 5(1), January 2010,1–22.
5. Manshath A, Balaji V, Sekar P, Elakkiya M,**Non – Existence of Skolem Mean Labeling for Five Star**, International Journal of Mathematical Combinatorics ISSN 1937 – 1055 Volume2, june 2017 pp 129 – 134.
6. Manshath A, Balaji V, Sekar P, Elakkiya M, **Further Result on Skolem Mean Labeling for Five Star**, Bulletin of Kerala Mathematics Association ISSN 0973 – 2721 Volume15, No.1 (2017, June) 85 – 93.
7. Manshath A, Balaji V, Sekar P, **Relaxed Skolem Mean Label for Five Star,** International Journal of Mathematics and its Application ISSN: 2347 – 1557 Volume 5,Issue 4 – D (2017) ,479 – 484.
8. Manshath A, Balaji V, Sekar P, **Relaxed Skolem Mean Labeling for Five Star**, International Journal of Mathematical Archive,ISSN 2229 – 5046 ,Volume – 8(7), 2017, 216 – 224
9. Manshath A, Balaji V, Sekar P, **Non Existence of Relaxed Skolem Mean Labeling for Star Graphs**, International Journal of Mathematical Archive ISSN 2229 – 5046, Volume – 8(10), 2017, 110 – 122.
10. Manshath A, Balaji V, Sekar P, **Non Existence of Skolem Mean Labeling for Four star Graph**, Mathematical Sciences International Research Journal ISSN 2278 – 8697, Volume 6 Issue 2 (2017) .