L-Fuzzy Meet Semi Almost Ideals

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Abstract
Algebraic structures like fuzzy sub rings of L-Fuzzy sets have been well studied in the case where the lattice $L$ is distributive. In this paper axioms the concepts can be generalised to the case where the lattice is not distributive. The aim of this current paper is to define the concept of $L$-fuzzy meet semi almost ideals and also Intutionistic $L$-fuzzy meet semi almost ideals. Investigate some theorems and examples.

Keywords: $L$-fuzzy Sets, $L$-fuzzy Almost Ideals, $L$-fuzzy Meet Semi Almost Ideals, Non-Distributive Lattice.

Introduction

Preliminaries
Let $X$ be a nonempty subset, $(L, \leq, \vee, \wedge)$ be a complete distributive lattice, which has least and greatest elements, say 0 and 1 respectively. Relevant definitions are recalled in this section.

Definition 2.1 Let $X$ be a nonempty set. A mapping by $\mu: X \rightarrow [0,1]$ is called a fuzzy subset of $X$.

Definition 2.2 Let $X$ be any non-empty set. A mapping $\mu: X \rightarrow L$ with $\mu(0)=1$ and $\mu(1)=0$ is called a $L$-fuzzy subset of $X$.

Definition 2.3 Let $A$ be a fuzzy meet semi Lattice. A fuzzy meet subsemi Lattice $\mu: X \rightarrow [0,1]$ is called fuzzy meet semi L- ideal of A if $\forall x, y \in A$, $\mu(x \wedge y) \geq \max\{\mu(x), \mu(y)\}$

Definition 2.4 Let $A$ be a fuzzy join semi Lattice. A fuzzy join subsemi Lattice $\mu: X \rightarrow [0,1]$ is called fuzzy join semi L- ideal of A if $\forall x, y \in A$, $\mu(x \vee y) \leq \min\{\mu(x), \mu(y)\}$

Definition 2.5 Let $R$ be a ring with unity. Let $L$ be a lattice $(L, \leq, \vee, \wedge)$ not necessarily distributive with least and greatest element 0 and 1 respectively. $\mu: R \rightarrow L$ with $\mu(0) = 1$ and $\mu(1) = 0$ is said to be $L$-fuzzy almost ideal if $\forall x, y \in R$
• \( \mu(x - y) \not\leq \mu(x) \land \mu(y) \)
• \( \mu(x y) \not\leq \mu(x) \) and \( \mu(x y) \not\leq \mu(y) \)

**Definition 2.6** Let \((L, \leq)\) be any lattice with an involutive order reversing operation \(N: L \to L\). Let \(X\) be any non-empty set. An Intuitionistic L-Fuzzy Set (ILFS) \(A\) in \(X\) is defined as an object of the form \(A = \{ <x, \mu(x), \nu(x) > \mid x \in X \} \), where the functions \(\mu: X \to L\) and \(\nu: X \to L\) define the degree of membership and the degree of non-membership respectively and for every \(x \in X\) satisfy \(\mu(x) \leq N(\nu(x))\).

**Definition 2.7** An Intuitionistic L-fuzzy subset \(A = \{ <x, \mu(x), \nu(x) > \mid x \in X \}\) of a ring \(R\) is called an Intuitionistic L-fuzzy subring of \(R\) if

\[
\begin{align*}
\mu_A(x - y) & \geq \mu_A(x) \land \mu_A(y) \\
\mu_A(xy) & \geq \mu_A(x) \lor \mu_A(y) \\
\nu_A(x - y) & \leq \nu_A(x) \lor \nu_A(y) \\
\nu_A(xy) & \leq \nu_A(x) \land \nu_A(y)
\end{align*}
\]

**Definition 2.8** Let \(A = \{ <x, \mu(x), \nu(x) > \mid x \in X \}\) be an Intuitionistic L-fuzzy ideal (ILFI) of \(R\) if

\[
\begin{align*}
\mu(x - y) & \geq \mu(x) \land \mu(y) \\
\mu(x y) & \geq \mu(x) \lor \mu(y) \\
\nu(x - y) & \leq \nu(x) \lor \nu(y) \\
\nu(x y) & \leq \nu(x) \land \nu(y)
\end{align*}
\]

**Remark 2.10** Consider \(\mu: X \to L\). If \(L\) is totally ordered then for all \(x, y \in R\), \(\mu(x)\) and \(\mu(y)\) are comparable. That is either \(\mu(x) \leq \mu(y)\) or \(\mu(x) = \mu(y)\) or \(\mu(x) \geq \mu(y)\). But if \(L\) is not totally ordered then are four possibilities \(\mu(x) \leq \mu(y)\) or \(\mu(x) = \mu(y)\) or \(\mu(x) \geq \mu(y)\) or \(\mu(x)\) and \(\mu(y)\) are not comparable. We use the notation \(\mu(x) \not\leq \mu(y)\) to mean, “\(\mu(x) \not\leq \mu(y)\) or \(\mu(x) = \mu(y)\) or \(\mu(x)\) and \(\mu(y)\) are not comparable”.

**L-Fuzzy Meet Semi Almost Ideals**

**Definition 3.1** Let \(R\) be a ring with unity. Let \(L\) be a lattice \((L, \leq, \lor, \land)\) not necessarily distributive with least and greatest element 0 and 1 respectively. \(\mu: R \to L\) with \(\mu(0) = 1\) and \(\mu(1) = 0\) is said to be L-fuzzy semi almost ideal if \(\mu(x y) \not\leq \mu(x) \lor \mu(y)\), for all \(x, y \in R\).

**Definition 3.2** Let \(R\) be a ring with unity. Let \(L\) be a lattice \((L, \leq, \lor, \land)\) not necessarily distributive with least and greatest element 0 and 1 respectively. \(\mu: R \to L\) with \(\mu(0) = 1\) and \(\mu(1) = 0\) is said to be L-fuzzy meet semi almost ideal if \(\mu(x \land y) \not\leq \mu(x) \lor \mu(y)\), for all \(x, y \in R\).

**Definition 3.3** Let \(R\) be a ring with unity. Let \(L\) be a lattice \((L, \leq, \lor, \land)\) not necessarily distributive with least and greatest element 0 and 1 respectively. \(\mu: R \to L\) with \(\mu(0) = 1\) and \(\mu(1) = 0\) is said to be Intuitionistic L-fuzzy meet semi almost ideal if \(\forall x, y \in R\)

\[
\begin{align*}
\mu(x \land y) & \not\leq \mu(x) \lor \mu(y) \\
\mu(x \land y) & \not\geq \mu(x) \land \mu(y)
\end{align*}
\]

**Example 3.4** The following is an example of a L-fuzzy meet semi almost ideal. Let \(R = \{0, a, b, c, 1\}\). Let \(L\) be a lattice \((L, \leq, \lor, \land)\) defined by below Hasse diagram. Note that \(L\) is not distributive. Define
μ: \( \mathbb{R} \rightarrow \mathbb{L} \) with \( \mu(x) = 1 \) as \( \mu(x) = \begin{cases} 1 & \text{if } x = 0.4 \\ a & \text{if } x = 0.5 \\ b & \text{if } x = 0.6 \\ c & \text{if } x = 0.7 \\ 0 & \text{if } x = 0.8 \end{cases} \)

Verification that \( \mu \) is a \( \mathbb{L} \)-fuzzy meet semi almost ideal can be summarized in the form of a table as follows

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>( \mu(x) \lor \mu(y) )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>c</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( \mu(x \land y) )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>( \mu(x) \lor \mu(y) )</td>
<td>1</td>
<td>a</td>
<td>1</td>
<td>1</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>( \mu(x \land y) )</td>
<td>1</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>0.6</td>
<td>( \mu(x) \lor \mu(y) )</td>
<td>1</td>
<td>1</td>
<td>b</td>
<td>1</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td>( \mu(x \land y) )</td>
<td>1</td>
<td>a</td>
<td>b</td>
<td>b</td>
<td>B</td>
</tr>
<tr>
<td>0.7</td>
<td>( \mu(x) \lor \mu(y) )</td>
<td>c</td>
<td>1</td>
<td>1</td>
<td>c</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( \mu(x \land y) )</td>
<td>a</td>
<td>1</td>
<td>b</td>
<td>a</td>
<td>c</td>
</tr>
<tr>
<td>0.8</td>
<td>( \mu(x) \lor \mu(y) )</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>( \mu(x \land y) )</td>
<td>1</td>
<td>c</td>
<td>1</td>
<td>c</td>
<td>a</td>
</tr>
</tbody>
</table>

**Example 3.5** Let \( \mathbb{L} \) be the lattice given by Hasse diagram in figure-I. Note that it is not distributive. N: \( \mathbb{L} \rightarrow \mathbb{L} \) is defined by \( N(0) = 1, N(a_i) = a_{3-i}, N(b) = b, N(c_i) = c_{3-i}, N(1) = 0 \).

Let \( A \) be an ILFS defined by \((\mu, \nu)\) where

\[
\begin{align*}
\mu(x) = 0, \quad \nu(x) &= 1 \quad \text{if } x \in \mathbb{Z} - \langle 3 \rangle - \langle 5 \rangle \\
\mu(x) = a_1, \quad \nu(x) &= a_1 \quad \text{if } x \in \langle 3 \rangle - \langle 3^2 \rangle - \langle 5 \rangle \\
\mu(x) = a_2, \quad \nu(x) &= a_1 \quad \text{if } x \in \langle 3^2 \rangle - \langle 3^3 \rangle - \langle 5 \rangle \\
\mu(x) = a_3, \quad \nu(x) &= a_1 \quad \text{if } x \in \langle 3^3 \rangle - \langle 3^4 \rangle - \langle 5 \rangle \\
\mu(x) = b = \nu(x) \quad &\text{if } x \in \langle 15 \rangle \\
\mu(x) = c_1, \quad \nu(x) &= c_1 \quad \text{if } x \in \langle 5 \rangle - \langle 5^2 \rangle - \langle 3 \rangle \\
\mu(x) = c_2, \quad \nu(x) &= c_1 \quad \text{if } x \in \langle 5^2 \rangle - \langle 5^3 \rangle - \langle 3 \rangle \\
\mu(x) = c_3, \quad \nu(x) &= c_1 \quad \text{if } x \in \langle 5^3 \rangle - \langle 5^4 \rangle - \langle 3 \rangle \\
\mu(x) = 1, \quad \nu(x) &= 0 \quad \text{if } x = 0
\end{align*}
\]

Figure I
By routine calculation we can verify that $A$ is intuitionistic $L$-fuzzy meet semi almost ideal.

**Theorem 3.6** The union of two intuitionistic $L$-fuzzy meet semi almost ideal is an intuitionistic $L$-fuzzy meet semi almost ideal.

**Theorem 3.7** The intersection of two intuitionistic $L$-fuzzy meet semi almost ideal is an intuitionistic $L$-fuzzy meet semi almost ideal.

**Theorem 3.8** The complement of two intuitionistic $L$-fuzzy meet semi almost ideal is an intuitionistic $L$-fuzzy meet semi almost ideal.

**Conclusion**

In this paper we have developed the concept of $L$-fuzzy meet semi almost ideal and also investigate the intuitionistic $L$-fuzzy meet semi almost ideal in the case of $L$ is not necessarily distributive. We have proved some theorems and examples.

**References**