ON SUPRA G*BΩ - CLOSED SETS IN SUPRA TOPOLOGICAL SPACES

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Abstract

The aim of this paper is to introduce the concepts of supra generalized star bω - closed sets and study their basic properties in supra topological spaces.

Keywords: *supra g*bω - closed sets and supra g*bω - open sets.*

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Introduction

Levine [3] introduced the concept of generalized closed sets in topological spaces. Andrijevic [1] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. In 1982, Hdeib [2] introduced the notions of ω - closed set. In 2012, A.Parvathi et al. [5] introduced g*bω - closed sets in topological spaces. In 1983, A.S.Mashhour et al [4] introduced the notion of supra topological spaces and studied S-S continuous functions and S* - continuous functions. In 2010, O.R.Sayed and Takashi Noiri [6] introduced supra b - open sets and supra b - continuity on topological spaces. In this paper, we introduce the concepts of supra generalized star bω - closed sets, supra generalized star bω - open sets and study their basic properties in supra topological spaces.

Preliminaries

Definition 1.1 [4, 6] A subfamily μ of X is said to be a supra topology on X if

- *X*, $\varphi \in \mu$
- if $A_i \in \mu$ for all i, then $\cup A_i \in \mu$

The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition 1.2 [4] The supra closure of a set A is defined as $cl^{\mu}(A)$ = ∩{B: B is supra closed and A ⊆ B} and the supra interior of a set A is defined as $int^{\mu}(A)$ = ∪{ B : B is supra open and A ⊇ B}.

Throughout this paper we shall denote by (X, μ) a supra topological space. For any subset A ⊆ X, $int^{\mu}(A)$ and $cl^{\mu}(A)$ denote the supra interior of A and the supra closure of A with respect to μ.

We shall require the following known definitions:

Definition 2.1 [4] Let (X, μ) be a topological spaces. A subset A of X is called

- *supra semi - open* if $A \subseteq cl^{\mu}(int^{\mu}(A))$ and *supra semi - closed* if $int^{\mu}(cl^{\mu}(A)) \subseteq A$
- *supra pre open* if $A \subseteq int^{\mu}(cl^{\mu}(A))$ and *supra pre closed* if $cl^{\mu}(int^{\mu}(A)) \subseteq A$
- *supra ~ - open* if $A \subseteq int^{\mu}(cl^{\mu}(int^{\mu}(A)))$ and *supra ~ - closed* if $cl^{\mu}(int^{\mu}(cl^{\mu}(A))) \subseteq A$
- *supra regular open* if $A = int^{\mu}(cl^{\mu}(A))$ and *supra regular closed* if $A = cl^{\mu}(int^{\mu}(A))$
- *supra b - open* if $A \subseteq cl^{\mu}(int^{\mu}(A)) \cup int^{\mu}(cl^{\mu}(A))$ and
  supra b - closed if $cl^{\mu}(int^{\mu}(A)) \cap int^{\mu}(cl^{\mu}(A)) \subseteq A$.

Let (X,μ) or simply X denote a supra topological space. For any subset A ⊆ X, the intersection of all supra semi closed (resp. supra pre closed, supra α - closed, supra regular closed, supra b - closed) sets containing A is called the *supra semi closure* (resp. *supra pre closure, supra α - closure, supra regular closure, supra b - closure*) of A, denoted by scl^{μ}(A) (resp. pcl^{μ}(A), αcl^{μ}(A), rcl^{μ}(A), bcl^{μ}(A)). The union of all semi open (resp. pre open, α-open, regular open, b-open) sets contained in A is called the *supra semi interior* (resp. *supra pre interior, supra α - interior, supra regular interior, supra b - interior*) of A, denoted by $sint^{\mu}$(A) (resp. $pint^{\mu}$(A), αint^{μ}(A), $rint^{\mu}$(A), $bint^{\mu}$(A)).

Definition 2.2 [4] Let (X, μ) be a supra topological space. A subset A of X is called

- *Supra generalized closed* (briefly *g^{μ} - closed*) if cl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra open in (X, μ).
- *Supra α generalized closed* (briefly *αg^{μ} - closed*) if αcl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra open in (X, μ).
- *Supra generalized semi closed* (briefly *gs^{μ} - closed*) if scl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra open in (X, μ).

Supra generalized star b omega - Closed Sets

Definition 3.1

A set A of a supra topological space (X, μ) is called ***supra generalized star b omega closed*** (briefly, g*bω^{μ} - closed) if bcl^{μ}(A) ⊆ U whenever A ⊆ U and U is supra gs - open in (X, τ).
The set of all g*bω^{μ} - closed sets in X is denoted by G*bω^{μ}C(X).

Example 3.2: Let X = {a, b, c} with the topology μ = {φ, X, {a, b}, {b, c}}. The subsets {a}, {b}, {c}, {a, b}, {b, c} and {a, c} are g*bω^{μ} - closed.

Theorem 3.3: Every supra closed set is g*bω^{μ} - closed.

Proof: Let A ⊆ U and U be gs^{μ} - open in X. Since A is supra closed in X and bcl^{μ}(A) ⊆ cl^{μ}(A), bcl^{μ}(A) ⊆ U. Therefore A is g*bω^{μ} - closed.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.4 Let X = {a, b, c} with the topology μ = {φ, X, {a, b}, {b, c}}. The subsets {a}, {b}, {c} and {a, c} are g*bω^{μ} - closed but not supra closed.

Theorem 3.5 Every supra semi closed set is g*bω^{μ} - closed.

Proof: Let A ⊆ U, where U is gs^{μ} - open. Since A is supra semi closed and bcl^{μ}(A) ⊆ scl^{μ}(A), bcl^{μ}(A) ⊆ U. Hence A is a g*bω^{μ} - closed set in X.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.6: In example 3.4, the subsets {b}, {a, b}, {b, c} and {a, c} are g*bω^{μ} - closed sets but not supra semi closed.

Theorem 3.7: Every supra α - closed set is g*bω^{μ} - closed.

Proof: Let A ⊆ U and U be gs^{μ}- open in X. Since A is supra α - closed in X, αcl^{μ} (A) = A ⊆ U and bcl^{μ}(A) ⊆ αcl^{μ}(A), bcl^{μ}(A) ⊆ U. Therefore A is g*bω^{μ} - closed.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.8: In example 3.4, the subsets {b}, {a, b}, {b, c} and {a, c} are g*bω^{μ} - closed sets but not supra α - closed.

Theorem 3.9: Every supra pre closed set is g*bω^{μ} - closed.

Proof: Let A ⊆ U and U be gs^{μ}- open in X. Since A is supra pre closed in X and bcl^{μ}(A) ⊆ pcl^{μ}(A), bcl^{μ}(A) ⊆ U. Therefore A is g*bω^{μ} - closed.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.10: In example 3.4, the subsets {a, b} and {b, c} are g*bω^{μ} - closed sets but not supra pre closed.

Theorem 3.11: Every supra regular closed set is g*bω^{μ} - closed.

Proof: Let A ⊆ U and U be gs^{μ}- open in X. Since A is supra regular closed in X and bcl^{μ}(A) ⊆ rcl^{μ}(A), bcl^{μ}(A) ⊆ U. Therefore A is g*bω^{μ} - closed.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.12: In example 3.4, the subsets {a}, {b}, {c}, {a, b} {a, c} and {b, c} are g*bω^{μ} - closed sets but not supra regular closed.

Theorem 3.13: Every supra b - closed set is g*bω^{μ} - closed.

Proof: Let A ⊆ U and U be gs^{μ}- open in X. Since A is supra b - closed in X, bcl^{μ} (A) = A ⊆ U. Therefore A is g*bω^{μ} - closed.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.14: In example 3.4, the subsets {a, b} and {b, c} are g*bω^{μ} - closed sets but not supra b - closed.

Remark 3.15: The following examples show that the concept of g^{μ} - closed and g*bω^{μ} - closed sets are independent.

Example 3.16: Let X = {a, b, c} with the topology μ = {φ, X, {a}, {a, b}}. The subset {a, c} is g^{μ} - closed but not g*bω^{μ} - closed.

Example 3.17: In example 3.16, the subset {b} is g*bω^{μ} - closed but not g^{μ} - closed.

Remark 3.18: The following examples show that the concept of αg^{μ} - closed and g*bω^{μ} - closed sets are independent.

Example 3.19: In example 3.16, the subset {a, c} is αg^{μ} - closed but not g*bω^{μ} - closed.

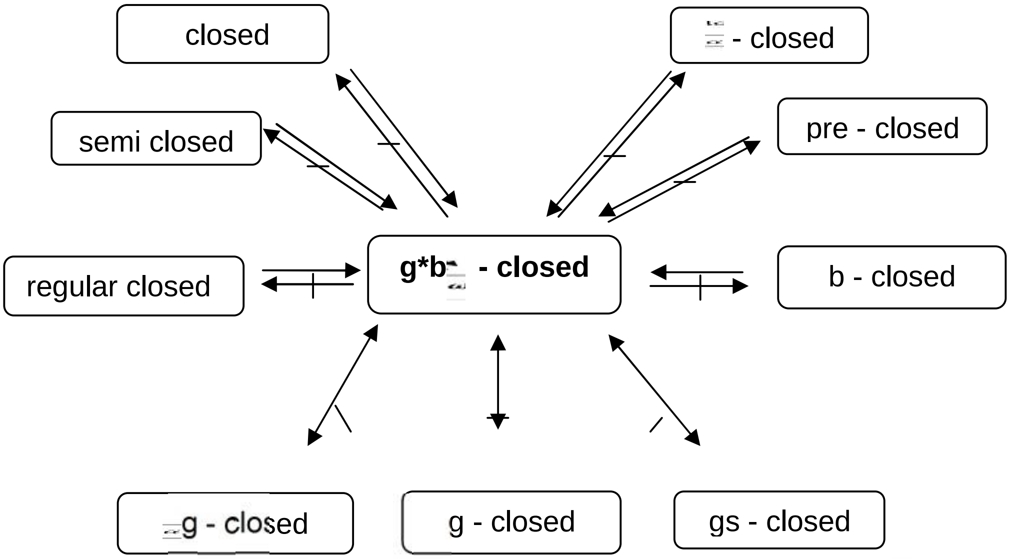
Example 3.20: Let X = {a, b, c} with the topology μ = {φ, X, {a}, {b}, {a, b}}. The subsets {a} and {b} are g*bω^{μ} - closed but not αg^{μ} - closed.

Remark 3.21: The following examples show that the concept of gs^{μ} - closed and g*bω^{μ} - closed sets are independent.

Example 3.22: In example 3.20, the subset {a, c} is gs^{μ} - closed but not g*bω^{μ} - closed.

Example 3.23: Let X = {a, b, c, d} with the topology μ = {φ, X, {a, b}, {a, b, c}, {a, b, d}. The subsets {a}, {b}, {a, c}, {a, d}, {b, c} and {b, d} are g*bω^{μ} - closed sets but not gs^{μ} - closed.

Remark 3.24 The following relation has been proved for g*bω^{μ} - closed sets.



Theorem 3.25 If A is gs^{μ} - open and g*bω^{μ} - closed then A is supra b - closed.

Proof: Suppose that A is gs^{μ} - open and g*bω^{μ} - closed. Since A ⊆ A and A is g*bω^{μ} - closed in X, bcl^{μ}(A) ⊆ A. But always A ⊆ bcl^{μ}(A). Therefore bcl(A) = A. Consequently A is supra b - closed.

Theorem 3.26 If A is g*bω^{μ} - closed and gs^{μ} - open and F is supra b - closed in X then A ∩ F is supra b - closed in X.

Proof: Since A is g*bω^{μ} - closed and gs^{μ} - open in X, we have A is supra b - closed (by Theorem 3.25). Since F is supra b - closed in X then A ∩ F is supra b - closed in X.

Theorem 3.27 The union of two g*bω^{μ} - closed sets is g*bω^{μ} - closed.

Proof: Let A and B are two g*bω^{μ} - closed sets in X. Let A ∪ B ⊆ U and U be supra gs - open in X. Since A and B are g*bω^{μ} - closed sets, bcl^{μ}(A) ⊆ U and bcl^{μ}(B) ⊆ U. Therefore [bcl^{μ}(A)] ∪ [bcl^{μ}(B)] ⊆ U. Since bcl^{μ}(A ∪ B) ⊆ U. Hence A ∪ B is g*bω^{μ} - closed.

Theorem 3.28 If A and B are g*bω^{μ} - closed sets then A ∩ B is g*bω^{μ} - closed.

Proof: Given that A and B are two g*bω^{μ} - closed sets in X. Let A ∩ B ⊆ U and U be supra gs - open in X. Since A and B are g*bω^{μ} - closed sets, bcl^{μ}(A) ⊆ U and bcl^{μ}(B) ⊆ U. Therefore [bcl^{μ}(A)] ∩ [bcl^{μ}(B)] ⊆ U. Since bcl^{μ}(A ∩ B) ⊂ [bcl^{μ}(A)] ∩ [bcl^{μ}(B)], bcl^{μ}(A ∩ B) ⊆ U Hence A ∩ B is g*bω^{μ} - closed.

The converse of the above theorem is not true in general as can be seen from the following example.

Example 3.29 Let X = {a, b, c, d} with the topology μ = {φ, X, {a, b}, {a, b, c}, {a, b, d}. Then Let A = {a} and B = {a, b}. Then A ∩ B = {a} is g*bω^{μ} - closed, A = {a} is g*bω^{μ} - closed but B = {a, b} is not g*bω^{μ} - closed.

Theorem 3.30 Let A be a subset of a topological space (X, μ). If A is g*bω^{μ} - closed then bcl^{μ}(A) \ A contains no nonempty gs^{μ}- closed set.

Proof: Suppose that A is g*bω^{μ} - closed. Let F be a gs^{μ} - closed set such that F $\subseteq bcl^{\mu}$(A) \ A. We shall show that F = φ. Since F $\subseteq bcl^{\mu}$(A) \ A, we have A $\subseteq$ F^c and F $\subseteq bcl^{\mu}$(A). Since F is a gs^{μ} - closed set, we have F^c is gs^{μ} - open. Since A is g*bω^{μ} - closed, we have bcl^{μ}(A) $\subseteq$ F^c. Thus F $\subseteq$ $[bcl^{\mu}(A)]^c$ = X \ [bcl^{μ}(A)]. Hence F $\subseteq$ [bcl^{μ}(A)] ∩ [X \ [bcl^{μ}(A)]] = φ. Therefore F = φ. Hence bcl^{μ}(A) \ A contains no nonempty gs^{μ} - closed sets.

Theorem 3.31 Let A be a g*bω^{μ} - closed set. Then A is supra b - closed in X if and only if bcl^{μ}(A) \ A is gs^{μ} - closed in X.

Proof: Suppose that A is g*bω^{μ} - closed. Let A be supra b - closed. Then bcl^{μ}(A) = A. Therefore bcl^{μ}(A) \ A = φ is gs - closed in X.

Conversely, suppose that A is g*bω^{μ} - closed and bcl^{μ}(A) \ A is gs^{μ} - closed. Since A is g*bω^{μ} - closed, bcl^{μ}(A) \ A contains no nonempty gs^{μ} - closed set (by Theorem 3.56). Since bcl^{μ}(A) \ A is gs^{μ} - closed, bcl^{μ}(A) \ A = φ. Then bcl^{μ}(A) = A. Hence A is supra b - closed.

Theorem 3.32 Let A and B be subsets such that A $\subseteq$ B $\subseteq bcl^{\mu}$(A). If A is g*bω^{μ} - closed then B is g*bω^{μ} - closed.

Proof: Let A and B be subsets such that A $\subseteq$ B $\subseteq bcl^{\mu}$(A). Suppose that A is g*bω^{μ} - closed. Let B $\subseteq$ U and U be gs^{μ} - open in X. Since A $\subseteq$ B and B $\subseteq$ U, A $\subseteq$ U. Since A is g*bω^{μ} - closed, bcl^{μ}(A) $\subseteq$ U. Since B $\subseteq bcl^{\mu}$(A), bcl^{μ}(B) $\subseteq bcl^{\mu}[bcl^{\mu}$(A)] = bcl^{μ}(A) $\subseteq$ U. Therefore B is g*bω^{μ} - closed.

Corollary 3.33 If A is g*bω^{μ} - closed and A $\subseteq$ B $\subseteq bcl^{\mu}$(A) then bcl^{μ}(B) \ B contains no nonempty gs^{μ} - closed set.

Supra generalized star b omega - Open Sets

Definition 4.1 A set A of a topological space (X, μ) is called ***supra generalized star b omega open*** (briefly, g*bω^{μ}- open) if and only if A^c is g*bω^{μ} - closed in X.

Theorem 4.2 A subset A of a topological space (X, μ) is g*bω^{μ} - open if and only if F $\subseteq$ $bint^{\mu}$ (A) whenever F $\subseteq$ A and F is gs^{μ} - closed in X.

Proof: Suppose that A is g*bω^{μ} - open. Let F $\subseteq$ A and F be gs^{μ} - closed. Then A^c $\subseteq$ F^c and F^c is gs^{μ} - open. Since A is g*bω^{μ} - open, A^c is g*bω^{μ} - closed. Hence $bcl^{\mu}(A^c)$ $\subseteq$ F^c. Since $bcl^{\mu}(A^c)$ = $[bint^{\mu}(A)]^c$, $[bint^{\mu}(A)]^c$ $\subseteq$ F^c. Hence F $\subseteq$ $bint^{\mu}$(A).

Conversely, suppose that F $\subseteq$ $bint^{\mu}$(A) whenever F $\subseteq$ A and F is gs^{μ} - closed in X. Let U be gs^{μ} - open in X and A^c $\subseteq$ U. Then U^c is gs^{μ} - closed and U^c $\subseteq$ A. Hence by assumption U^c $\subseteq$ $bint^{\mu}$(A) Therefore $[bint^{\mu}(A)]^c$ $\subseteq$ U. That is $bcl^{\mu}(A^c)$ $\subseteq$ U. Therefore A^c is g*bω^{μ} - closed. Hence A is g*bω^{μ} - open.

Theorem 4.3 If a subset A is g*bω^{μ} - closed in X then bcl^{μ}(A) \ A is g*bω^{μ} - open.

Proof: Suppose that A is g*bω^{μ} - closed in X. Let F $\subseteq$ bcl^{μ}(A) \ A and F be gs^{μ} - closed. Since A is g*bω^{μ} - closed, bcl^{μ}(A) \ A does not contain nonempty gs^{μ} - closed sets (by Theorem 3.30). Since F $\subseteq$ bcl^{μ}(A) \ A, F = φ. Since φ $\subseteq$ $bint^{\mu}[bcl^{\mu}$(A) \ A], F $\subseteq$ $bint^{\mu}[bcl^{\mu}$(A) \ A] Hence bcl^{μ}(A) \ A is g*bω^{μ} - open.

Theorem 4.4 For each x ∈ X, the singleton {x} is either gs^{μ} - closed or g*bω^{μ} - open.

Proof: Let x ∈ X and suppose that {x} is not gs^{μ} - closed. Then X \ {x} is not gs^{μ} -open. Consequently, X is the only gs^{μ} - open set containing the set X \ {x}. Therefore X \ {x} is g*bω^{μ} - closed. Hence {x} is g*bω^{μ} - open.

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