# MAGIC SQUARES

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**Abstract**

Magic squares have intrigued people for thousands of years and in ancient times they were thought to be connected with the supernatural and hence, magical. Today, we might still think of them as being magical, for the sum of each row, column and diagonal is a constant, the magic constant. The problem of construction is twofold. An algorithm which works for odd order squares will not work for even order squares without the further addition of another algorithm. Odd magic squares are fairly easily constructed using the either the Siamese (sometimes called de la Loubere's, or the Staircase method), the Lozenge, or the de Meziriac's methods.

**Keywords:** Magic square, de Meziriac's method, Lozenge Method, Eric's treasure, Pyramid method, de la Loubere's

Extended Pyramid method or diagonals. This method consists of three steps:

1. Draw a pyramid on each side of the magic square. The pyramid should have two less squares on its base than the number of squares on the side of the magic square. This creates a square standing on a vertex.
2. Sequentially place the numbers 1 to $n^2$ of the $n$ x $n$ magic square in the diagonals as shown in Figures 1 and 2.
3. Relocate any number not in the $n$ x $n$ square (that appears in the pyramids you added) to the opposite hole inside the square (shaded).

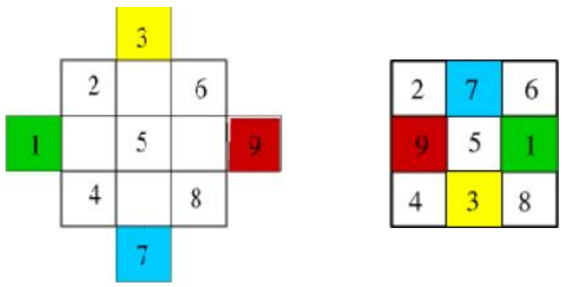


**Figure 1**

The same Pyramid method can be used for any odd order magic square as shown below for the 5x5 square in Figure 2.

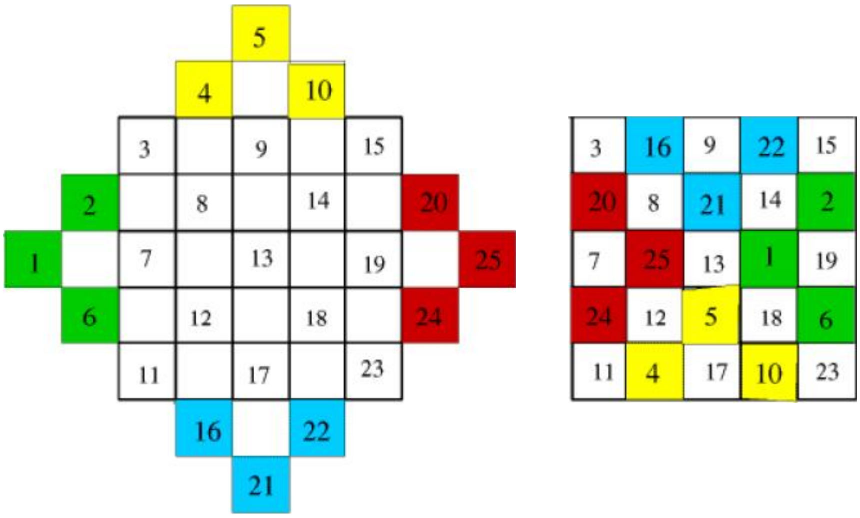


**Figure 2**

We can use some properties of magic squares to construct more squares from the manufactured squares above; *e.g.*

1. A magic square will remain magic if any number is added to every number of a magic square.
2. A magic square will remain magic if any number multiplies every number of a magic square.
3. A magic square will remain magic if two rows, or columns, equidistant from the centre are interchanged.
4. An even order magic square ( $n$ x $n$ where $n$ is even) will remain magic if the quadrants are interchanged.
5. An odd order magic square will remain magic if the partial quadrants and the row is interchanged.

Constructing the even order magic squares does present more of a challenge. There are many different ways, which can be studied through "Eric's treasure trove of Mathematics "; at least as a starting point.

A few magic squares have been given below. Trace out the pattern.

**Magic Square 12 x 12**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 144 | 2 | 3 | 141 | 136 | 10 | 11 | 133 | 128 | 18 | 19 | 125 |
| 5 | 139 | 138 | 8 | 13 | 131 | 130 | 16 | 21 | 123 | 122 | 24 |
| 137 | 7 | 6 | 140 | 1250 | 15 | 14 | 132 | 121 | 23 | 22 | 124 |
| 4 | 142 | 143 | 1 | 12 | 134 | 135 | 9 | 20 | 125 | 127 | 17 |
| 120 | 26 | 27 | 117 | 112 | 34 | 35 | 109 | 104 | 42 | 43 | 101 |
| 29 | 115 | 114 | 32 | 37 | 107 | 106 | 40 | 45 | 99 | 98 | 48 |
| 113 | 31 | 30 | 116 | 105 | 39 | 38 | 108 | 97 | 47 | 46 | 100 |
| 28 | 118 | 119 | 25 | 36 | 110 | 111 | 33 | 44 | 102 | 103 | 41 |
| 96 | 50 | 51 | 93 | 88 | 58 | 59 | 85 | 80 | 66 | 67 | 77 |
| 53 | 91 | 90 | 56 | 61 | 83 | 82 | 64 | 69 | 75 | 74 | 72 |
| 89 | 55 | 54 | 92 | 61 | 63 | 62 | 64 | 73 | 71 | 70 | 76 |
| 52 | 94 | 95 | 49 | 60 | 86 | 87 | 57 | 68 | 78 | 79 | 65 |

**Magic Square 9 x 9**

**Total = 999**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 117 | 128 | 139 | 150 | 71 | 82 | 93 | 104 | 115 |
| 127 | 138 | 149 | 79 | 81 | 92 | 103 | 114 | 116 |
| 137 | 148 | 78 | 80 | 91 | 102 | 113 | 124 | 126 |
| 147 | 77 | 88 | 90 | 101 | 112 | 123 | 125 | 136 |
| 76 | 87 | 89 | 100 | 111 | 122 | 133 | 135 | 146 |
| 86 | 97 | 99 | 110 | 121 | 132 | 134 | 145 | 75 |
| 96 | 98 | 109 | 120 | 131 | 142 | 144 | 74 | 85 |
| 106 | 108 | 119 | 130 | 141 | 143 | 73 | 84 | 95 |
| 107 | 118 | 129 | 140 | 151 | 72 | 83 | 94 | 105 |

**Magic Square 9 x 9**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 123 | 145 | 167 | 190 | 31 | 53 | 75 | 97 | 119 |
| 143 | 166 | 187 | 47 | 51 | 73 | 95 | 117 | 121 |
| 163 | 185 | 45 | 49 | 71 | 93 | 115 | 138 | 141 |
| 183 | 43 | 65 | 69 | 91 | 114 | 135 | 139 | 161 |
| 41 | 63 | 67 | 89 | 111 | 133 | 155 | 159 | 182 |
| 61 | 83 | 87 | 109 | 131 | 153 | 158 | 179 | 39 |
| 81 | 85 | 107 | 129 | 152 | 173 | 177 | 37 | 59 |
| 101 | 105 | 128 | 149 | 171 | 175 | 35 | 57 | 79 |
| 104 | 125 | 147 | 169 | 191 | 33 | 55 | 77 | 99 |

**Magic Square 8 x 8**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 64 | 63 | 3 | 4 | 5 | 6 | 58 | 57 |
| 1 | 2 | 62 | 61 | 60 | 59 | 7 | 8 |
| 56 | 55 | 11 | 12 | 13 | 14 | 50 | 49 |
| 9 | 10 | 54 | 53 | 52 | 51 | 15 | 16 |
| 48 | 47 | 19 | 20 | 21 | 22 | 42 | 41 |
| 17 | 18 | 46 | 45 | 44 | 43 | 23 | 24 |
| 40 | 39 | 27 | 28 | 29 | 30 | 34 | 33 |
| 25 | 26 | 38 | 37 | 36 | 35 | 31 | 32 |

**Magic Square 6 x 6**

| | | | | | |
|---|---|---|---|---|---|
| 8 | 28 | 33 | 17 | 10 | 15 |
| 30 | 5 | 34 | 12 | 14 | 16 |
| 4 | 36 | 29 | 13 | 18 | 11 |
| 35 | 1 | 6 | 26 | 19 | 24 |
| 3 | 32 | 7 | 21 | 23 | 25 |
| 31 | 9 | 2 | 22 | 27 | 20 |

**Magic Square 5 x 5**

| | | | | |
|---|---|---|---|---|
| 24 | 31 | 8 | 15 | 22 |
| 30 | 12 | 14 | 21 | 23 |
| 11 | 13 | 20 | 27 | 29 |
| 17 | 19 | 26 | 28 | 10 |
| 18 | 25 | 32 | 9 | 16 |

**Magic Square 4 x 4**
**Using Prime Number T = 180**

| | | | |
|---|---|---|---|
| 67 | 1 | 83 | 29 |
| 59 | 53 | 61 | 7 |
| 11 | 47 | 19 | 103 |
| 43 | 79 | 17 | 41 |

**Magic Square 4 x 4**

| | | | |
|---|---|---|---|
| 49 | 2 | 3 | 46 |
| 5 | 44 | 43 | 8 |
| 42 | 7 | 6 | 45 |
| 4 | 47 | 48 | 1 |

**Magic Square 4 x 4**
**T = 99**

| | | | |
|---|---|---|---|
| 49 | 3 | 4 | 43 |
| 9 | 38 | 37 | 15 |
| 35 | 13 | 11 | 40 |
| 6 | 45 | 47 | 1 |

**Magic Square 4 x 4**
**T = 101**

| | | | |
|---|---|---|---|
| 49 | 3 | 6 | 43 |
| 9 | 40 | 37 | 15 |
| 35 | 13 | 11 | 42 |
| 8 | 45 | 47 | 1 |

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